SANDIA REPORT

SAND2006-0068
Unlimited Release
Printed March 2006

A Framework for Modeling the Impacts of Terrorism on Confidence and the Economy

James Sprigg, George Backus, Rich Pryor, John Siirola, Gene Selzler, Craig Jorgensen, Alex Slepoy, Mark Ehlen, Tim Trucano, Michael Hand, and Paul Paez

Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550

Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000

Sandia National Laboratories

Issued by Sandia National Laboratories, operated for the United States Department of Energy by Sandia Corporation.

Available to DOE and DOE contractors from
U.S. Department of Energy
Office of Scientific and Technical Information
P.O. Box 62
Oak Ridge, TN 37831

Telephone: (865)576-8401
Facsimile: (865)576-5728
E-Mail: reports@adonis.osti.gov
Online ordering: http://www.osti.gov/bridge

Available to the public from
U.S. Department of Commerce
National Technical Information Service
5285 Port Royal Rd
Springfield, VA 22161

Telephone: (800)553-6847
Facsimile: (703)605-6900
E-Mail: orders@ntis.fedworld.gov
Online order: http://www.ntis.gov/help/ordermethods.asp?loc=7-4-0#online

SAND2006-0068
Unlimited Release
Printed March 2006

A Framework for Modeling the Impacts of Terrorism on Confidence and the Economy

James A. Sprigg Jr.
George A. Backus
Richard J. Pryor
John Siirola
Craig R. Jorgensen
Alexander Slepoy
Mark A. Ehlen
Tim G. Trucano

Sandia National Laboratories
PO Box 5800, MO 0370, Albuquerque, NM 87185-0370

Michael S. Hand
Paul J. Paez

University of New Mexico
Economics: MSC 05 3060, Albuquerque, NM 87131-0001

Gene Selzler

Strikewire Technologies
368 McCaslin Blvd, #115, Louisville, CO 80027

Abstract

This report presents the culmination of investigations into the interaction between expectations, real economic activities, and the potential impacts to confidence and the economy. We review pertinent theory and empirical findings. Our investigations employ both rigorous economic analysis and computation agent-based simulation. We implement our final model within a general framework for the visual design and execution of agent-based simulations.

Acknowledgments

We thank Jennifer Nelson and David Womble for helpful comments and suggestions.

Contents

Acknowledgments 4
1 Introduction 9
2 Theoretical and Empirical Underpinnings 12
2.1 Psychological Economics 12
2.2 Life-Cycle Theory and Permanent Income Hypothesis 13
2.3 Empirical Testing of PIH 15
2.4 Forecasting 15
2.5 Implications 17
3 Foundational Exploratory Simulations 18
3.1 Leisure Effects and Employment 18
3.1.1 Households 18
3.1.2 Labor Supply 19
3.1.3 Goods Demand 20
3.1.4 Firms 20
3.1.5 Market Equilibrium 21
3.1.6 Simulation Parameters and Mechanics 22
3.1.7 Simulation Results: Monopoly versus Oligopoly 23
3.1.8 Discussion 25
3.2 Life-Cycle Implications for Disruption and Recovery 27
3.2.1 Life-Cycle Hypothesis 27
3.2.2 Savings, Retirement, and the Bond Market 28
3.2.3 Market Equilibrium 29
3.2.4 Simulation Parameters and Mechanics 29
3.2.5 Simulation Results 31
3.2.6 Event Study of an Output Disruption with Finite Horizons 36
3.3 Remarks 40
4 The Analytic Model 41
4.1 Household Consumption and Savings 41
4.2 Firms and Nash Equilibrium 43
5 Agent-Based Simulation 44
5.1 Simulation Mechanics and Parameters 45
5.1.1 Bank 45
5.1.2 Households 45
5.1.3 Firms 46
5.2 Results 47
5.2.1 Set #1: Fixed Labor 47
5.2.2 Set #2: Adaptive Labor Scaling 50
5.3 Conclusions 53
6 An Agent-Based Modeling Framework 54
6.1 Characteristics of a General Modeling Framework 54
6.1.1 Modular 54
6.1.2 Hierarchical 54
6.1.3 Visual Interface 54
6.2 Features of a General Modeling Framework 55

6.2.1 Network Flow ... 55
6.2.2 Messaging ... 55
6.2.3 Cloning ... 55
6.2.4 Spontaneous Restructuring ... 55
6.3 Economic Simulation in the General Framework ... 55
6.3.1 Visual Interface ... 55
6.3.2 Modularization ... 56
6.3.3 Hierarchy ... 57
6.3.4 Summary ... 59
7 Remarks ... 60
8 References ... 61

Figures

Figure 1. Labor supply function ... 20
Figure 2. Competitive case: Total employment. ... 23
Figure 3. Competitive case: Average price. ... 23
Figure 4. Competitive case: Firm profits. ... 24
Figure 5. Competitive case: Units consumed. ... 24
Figure 6. Competitive case: Average units consumed. ... 25
Figure 7. Variability in firms' profits: Monopoly and competitive cases ... 26
Figure 8. Infinite horizon: deviation in bond holdings. ... 32
Figure 9. Infinite horizon: total employment. ... 33
Figure 10. Infinite horizon: market price of goods. ... 33
Figure 11. Infinite horizon: firm profits. ... 34
Figure 12. Infinite horizon: units consumed. ... 34
Figure 13. Finite horizon: equilibrium employment. ... 35
Figure 14. Finite horizon: total employment. ... 35
Figure 15. Finite horizon: market price of goods. ... 36
Figure 16. Finite horizon: deviation in bond holdings. ... 36
Figure 17. Event study of total employment. ... 37
Figure 18. Event study of firm profits. ... 38
Figure 19. Event study of household consumption. ... 38
Figure 20. Event study of deviation in bond holdings. ... 39
Figure 21. Event study of career households' budget for bonds ... 39
Figure 22. Event study of career households' target bond holdings. ... 40
Figure 23. Nash Equilibrium Price Formation Corresponding to Five Competing Firms with Different Labor Shares (i.e. numbers of employees). ... 49
Figure 24. Price Formation with Fixed versus Adjusted Labor Corresponding to Five Firms with Different Initial Labor Shares ... 51
Figure 25. Unemployment and Consumer Confidence ... 52
Figure 26. Visual Interface showing the top layer of Economic Simulation ... 56
Figure 27. Model Loop containing Economic Simulation ... 57
Figure 28. A Clone Wrapper for Household Agents ... 58
Figure 29. Settings for the "age index" input parameter. ... 59

Tables

Table 1. Summary of Findings for Confidence-Related Studies 16
Table 2. Simulation Parameters 22
Table 3. Simulation Parameters 31

Intentionally Left Blank

A Framework for Modeling the Impacts of Terrorism on Confidence and the Economy

1 Introduction

The national homeland-security agenda includes achieving a comprehensive understanding of the potential economic effects of terrorist acts. In addition to the direct impacts of an attack, such events might also incite overreactions by firms and households to the detriment of the economy. These secondary effects relate to a broader set of issues concerning the influence of information and perception on economic decision makers. For example, one might ask "How might households alter their consumption rates in response to a terrorist act?", and "How might those decisions affect the pricing and employment decisions of firms?"

Bird (2002) points out that "many commentators have suggested that the world changed on 11 September 2001. The terrorist acts in New York and Washington made the future more uncertain." Just one month after the 9/11 attacks, John Virgo, Executive Vice President of the International Atlantic Economic Society, chaired the *September 11th Panel Discussion* of the 52nd International Atlantic Economic Conference. Virgo observed that consumer confidence for September was at a five year low and in October slid to nearly an eight year low. He also observed that October [employment] figures showed the largest one-month decline in more than 21 years. "This makes it difficult for the rate cuts by the Federal Reserve to have the desired impact" (Virgo 2001 p355). Virgo reminds us that the economy was already slowing down before September 11th. The Federal funds rate had been cut 7 times in 2001 before the attack and three more times by November. At a Fed funds rate of only 2%, many economists worried that the Fed was running out of room to use monetary policy.

The issue of uncertainty undoubtedly has broad implications for public policy, not the least of which will be those of the Department of Homeland Security (DHS). Indeed, Homeland Security Secretary Tom Ridge stated in a speech at the New York Federal Reserve on July 8, 2003 that "Safeguarding the integrity of America's financial systems is a key part of homeland security." Clearly, DHS must understand the potential effects of terrorist events on economic confidence and financial markets in order to make comprehensive comparisons of the relative effectiveness of mitigation and response strategies.

Confidence and financial markets are governed largely by the economic forces at work within the consumption and savings decisions of discrete microeconomic agents, such as firms, households, and financial intermediaries. Economists provide a rigorous framework for investigating these forces, based largely on the groundbreaking work of Milton Friedman, Franco Modigliani, et al in the 1950s and 60s and culminating in hypotheses that form the foundation for much of our nation's monetary and fiscal policy. However, modern empirical research finds that actual consumer expenditures are more sensitive to current income than forecast by these hypotheses, suggesting the need for models of consumer confidence and precautionary savings. These issues have clear implications for modeling the effects of terrorist attacks on confidence and financial markets.

The conventional interplay between rigorous theory and statistical empirics provides insight into these questions, but often focuses on the properties of equilibria, without exploring conditions under which a system will or will not converge from a disequilibrium state to an “expected” equilibrium. Fortunately, the aforementioned theoretic framework integrates nicely into the discrete agent-based simulation framework, providing a powerful algorithmic foundation upon which to incorporate discrete models of confidence. We propose agent-based simulation as a viable extension to conventional methods, allowing for the adoption of established economic principles to provide a verifiable foundation for exploratory economic models.

This is the sixth in a series of reports describing our model and findings, and the first to define the model in a visual hierarchical modeling framework.

The first report (Sprigg, Jorgensen, and Pryor 2004) described a general approach and development strategy. It identified relevant economics principles and computations methods to be incorporated into simulation.

The second (Sprigg and Ehlen 2004) report modeled how microeconomic firms and employment adjust endogenously to changes in demand and in the number of firms. That report used agent-based simulations to demonstrate that, when compared to the case of monopoly, multiple-firm economies converge toward the competitive equilibria typified by lower prices and higher output and employment, but also suffer from market noise stemming from consumer churn.

The third report (Sprigg 2004) introduced life cycles and a simple bond market into the agent-based model, and demonstrated that labor, product, and bond markets converge to calculated equilibriums in accordance with a life-cycle hypothesis (LCH). Various simulations were conducted under different assumption to validate the model against accepted economic principle. This report also included an event study of an output disruption, and found that the disruption cascades through all sectors affecting employment, consumption, and savings. Following the event, the simulation re-converges, with all markets returning to equilibrium at about the same time.

The fourth report (Hand et al 2005) surveyed the relevant economic literature pertaining to economic confidence, including rational expectations, life-cycle theory, and the permanent income hypothesis. It also surveyed empirical work, which suggests that these conventional economic principles, alone, are insufficient to forecast or explain the volatility in observed financial behavior. Specifically, empirical studies suggest that consumption is more sensitive to current income than suggested under the permanent income hypothesis, which raises questions regarding expectations for future income, risk aversion, and the role of economic confidence measures.

The fifth report (Sprigg and Ehlen 2005) introduced a banking system and a more in-depth analytical model to establish the conditions of steady-state economic equilibrium. The corresponding economic was shown to converge to and maintain its Nash equilibrium in the goods market and financial optima in the financial market.

This report extends the previous work by adding enhanced price-discovery dynamics and is arranged as follows. Section 2 explores the primary theoretical and empirical findings and conclusions to establish a foundation for the scope and direction of this effort. Section 3 presents increasingly complex simulations that systematic explore pertinent economic principles and relationships. Section 4 presents an analytical model describing the expected dynamic equilibrium conditions for a system of interrelated labor, goods, and banking markets. Section 5 describes a simulation of the prescribed model and presents the corresponding computational results. Section 6 shows the economic simulation as implemented in software that uses a general agent-based framework for the visual design of modular hierarchical models.

2 Theoretical and Empirical Underpinnings

A variety of economic principles must be considered when designing a simulation of confidence and the economy. Of particular interest in economics research, no more so than after 9/11, is how general uncertainty about world events affects individual consumption decisions. The popular press often cites consumer confidence measures as indices of risk perception, implying that aggregate consumption reflects these measures in some way. But do these indices measure risk perceptions in a way that is useful for economic analysis? Or do they reflect agents' knowledge of more traditional economic indicators? Before any link between consumer confidence measures and consumption can be developed, an understanding of the various theories of how uncertainty affects consumption is necessary.

2.1 Psychological Economics

An explanation of uncertainty and its role in determining consumer behavior began with George Katona (1975) of the University of Michigan's Institute for Social Research. A psychologist by training, Katona developed a survey that asked people about their current financial situation, the business climate, and their expectations of their job and earnings prospects in the future. Using this survey, Katona created an index of consumer confidence; higher values indicated that the economic situation is good and projected by individuals to get better; lower values show pessimism about the economy and the future. Such an index is consistent with the expected utility theories discussed in the previous section; changes in the index value may reflect changes in perceptions of probabilities of future outcomes.

The survey and confidence index reflected Katona's view (and a budding vein of research) that psychology plays a role in individual consumption decisions. The psychological impact on consumption can be summed up by saying that consumption is determined by agents' ability and willingness to buy. At its heart, psychological economics recognizes that agents are not "marionettes pushed around by external forces," (Katona 1975 p8). Rather than acting as automatons to changes in prices, wages, and income, agents determine their level (and type) of consumption based on "attitudes, aspirations, and expectations" (Katz 1972 p65).

This thesis stems from the notion that consumption as a function of only prices and income is not well suited to the affluent post-World War II American economy. Consumers after WWII found themselves with discretionary income and the power to choose among a broad range of products and investments. Thus, consumption in the U.S. reflected tastes, preferences, and the willingness-to-buy as much as prices and income.

A specific representation of the consumption function was not developed as a result of Katona's work. This appears to be a deliberate implication of joining psychology and economics; if behavior is governed by laws and is measurable, then the empirical observation of behavior should uncover those laws. Instead of theoretically reasoning and testing the hypothesis that a change in income will create predictable changes in consumption, the psychological economist would observe agents' behavior (and perhaps motives or perception) in response to income changes and then reason a theory to explain the behavior.

2.2 Life-Cycle Theory and Permanent Income Hypothesis

Although Katona's work provided a framework for understanding the psychology of choice, the lack of a formal model of consumption would not do for most economists. In the years following the publication of John Maynard Keynes' *General Theory* in 1936, an impressive literature was devoted to his hypothesis of the relationship between income and consumption. While this literature did not explicitly recognize the psychological forces involved, economists increasingly recognized that a simple model of consumption, one based on current income and prices was inadequate. Synthesizing this literature, Franco Modigliani and Richard Brumberg developed a model of consumption where individuals derive utility from current and future consumption, and consumption in the current period is a function of current income, expected income, and the individual's initial set of assets. This model, known as the life-cycle hypothesis (LCH) model describes household behavior as an attempt to smooth consumption patterns over one's lifetime somewhat independent of current levels of income; households do this by borrowing, saving, and lending. The model is typically represented as the following constrained maximization problem:

$$\begin{aligned} &\max U_{t=0} = \sum_{t=0}^{T(t)} \delta(U_t(C_t)) \\ &s.t. \sum_{t=0}^{T(t)} \rho(C_t) = B_{t=0} + \sum_{t=0}^{L(t)} \rho(Y_t), \end{aligned} \tag{1}$$

where C_t and Y_t are respectively the levels of consumption and income in period t, $U_t(C_t)$ is the *utility* received from consumption in time period t, B_0 is an initial wealth endowment, and $\delta(\cdot)$ and $\rho(\cdot)$ are discount functions; $\delta(\cdot)$ discounts the value of future utility according to the household's internal time preference, and $\rho(\cdot)$ discounts the value of future consumption and income according to the market interest rate. Also, $L(t) \leq T(t)$, where $L(t)$ denotes the number of remaining periods in which the household can work in the labor market to earn income, and $T(t)$ denotes the number of remaining periods in the household's life cycle.

The main implication is that consumption and income need not coincide in any given period; individuals have motives to save or dissave. The primary purpose of saving is to cushion against future income fluctuations, though the inclusion of uncertainty in the model would introduce two additional motives. First, individuals might have a precautionary motive, or a "desire to accumulate assets…to meet possible emergencies," (Modigliani and Brumberg, 1955 p392). Second, individuals may feel the need to acquire an equity stake in durables in the face of uncertainty. In this case, individuals would save in anticipation of a consumer durables purchase so that less debt would be incurred for the purchase. If consumers are uncertain of their ability to repay debts in the future, paying a larger share out-of-pocket would reduce the impact of such uncertainty.

Modigliani and Brumberg find that Keynes' hypothesis – that individuals will increase consumption in proportion to an increase in income – is explained by their model. Further, they contend that the rate of savings is constant across levels of income.

Although a specific role for expected income appears in the model, no discussion is made of how those expectations are formed or how consumption changes in response to a change in expected income. This shortcoming is addressed to some degree by the Permanent Income Hypothesis (PIH) and its variant the Rational Expectations Permanent Income Hypothesis (REPIH).

The basic idea of PIH is that current period consumption is determined by lifetime resources, not measured income at a given point in time. Permanent income, then, is the annuity value of lifetime net worth. Consumption in a given time period is proportional to permanent income. Incorporating rational expectations explicitly states that agents' expectations of future income are formed using all available information and with full knowledge of the structure of the economy.

Under PIH, individuals determine their permanent income, and thus consumption, by evaluating their expectations of future income. Uncertainty exists in the path of future income, but the assumption of rational expectations gives rise to certainty equivalence with respect to contemporaneous consumption decisions; agents do not know the nature of future income shocks, so decisions are made as though the uncertainty does not exist.

An implication of certainty equivalence is that only changes in expected income can change permanent income (and thus consumption) in the current period. For example, an agent who observes a higher income in the current period (perhaps the result of a wage increase) might expect that the higher level of income will occur in future periods, changing their expected future income. As a result, consumption would contemporaneously change.

The response of consumption to expectations provides the most logical and developed role for consumer confidence in determining consumption. The strength of this role depends on the information consumer confidence indices contain about expected income. If consumer confidence adequately summarizes agents' beliefs about future income, then its role is consistent with PIH. But if consumer confidence predicts current consumption, then it is not consistent with PIH; recall: consumption can only change as a result of changes in expected income.

Several assumptions of PIH require explicit treatment. Foremost, households are assumed to be forward-looking. Second, credit markets must be perfect; agents must be able to borrow and save against future income. If an individual expects a higher future income, they might consume more now by borrowing against future income. Credit market constraints will inhibit this process and consumption would not increase until future income is realized. Thus the link between expectations and consumption would be broken, and PIH would not be valid.

Finally, the interest rate is assumed to be constant over time. This eliminates changes in consumption due the risk of interest rate fluctuations, though these concepts can be added to the model.

2.3 Empirical Testing of PIH

Though the PIH theory of consumption has been well fleshed-out, most studies reject the pure PIH or REPIH for a myriad of reasons. An oft-cited reason for rejection is Flavin's (1981) "excess sensitivity to current income." An implication of Flavin's consumption model is that the revision in permanent income is proportional to the observed error of forecasted income (i.e. "innovation in current income"). REPIH is then tested by whether or not the marginal propensity to consume (MPC) out of current income is zero (i.e. MPC = 0 implies acceptance of REPIH). Flavin finds that the MPC is significantly not equal to zero, and so determines that consumption is excessively sensitive to current income, thus rejecting REPIH.

Flavin's findings are supported by Campbell and Mankiw (1990), who find in addition that the rejection of REPIH cannot be explained by relaxing the assumptions of constant interest rate or separability of the consumers' utility function.

Acemoglu and Scott (1994) also reject REPIH, but do so by incorporating a measure of consumer confidence. They use confidence as a proxy for individuals' expectations and find that confidence is a leading indicator of consumption changes. This finding is inconsistent with REPIH, where only changes in income expectations can change consumption.

Acemoglu and Scott's work is also noteworthy because it defines a different role for consumer confidence measures. In their attempt to explain the rejection of REPIH, they test whether imperfect capital markets or the precautionary saving motive might account for confidence predicting consumption. Precautionary saving (and not imperfect capital markets) is found to explain the relationship. Their finding suggests a role for confidence measures, not for modeling income expectations, but for explaining risk aversion.

2.4 Forecasting

The exact link between uncertainty, expectations, and consumption is still up for debate, but the impact of consumer confidence on the economy and consumer expenditures has been thoroughly investigated. A prodigious literature is dedicated to the question of whether consumer confidence measures contain information important for economic forecasting and predicting consumer expenditures. Most of these studies, using various econometric models, test whether consumer confidence by itself has predictive content or whether adding consumer confidence improves the predictive ability of forecasts based on leading economic indicators.

The literature is divided on the predictive content of consumer confidence. When confidence measures are found to be significant predictors of expenditures, they often add little in terms of predictive ability. Some studies find no predictive value in confidence measures, or that consumer confidence, on it own, is a poor predictor of consumption. Table 1 summarizes some of the findings in the literature.

Table 1. Summary of Findings for Confidence-Related Studies

Study	Did consumer confidence improve forecasting model?	Is consumer confidence modeled alone or in conjunction with other variables?
Garner (1991)	Not a good predictor of consumption.	Both
Batchlor and Dua (1998)	Would have helped predict 1991 recession but not 1982 recession.	With other variables
Eppright, Arguca, and Huth (1998)	Consumer confidence has some predictive power not in other economic indicators	With other variables
Howrey (2001)	Consumer confidence modestly increases accuracy of forecasting models.	Both
Desroches and Gosselin (2002)	Not helpful in predicting consumer spending.	Modeled alone
Garner (2002)	Improved forecasting models slightly.	With other variables

These findings are not heartening for researchers seeking a role for consumer confidence in forecasting shocks to the economy, but additional findings provide some hope. Batchelor and Dua (1998) find that consumer confidence may contain important non-economic information. For example, they find that including a measure of consumer confidence in forecasting models would have help predict the 1991 recession, but not the 1982 recession. They hypothesize that this reveals the “special circumstances” surrounding the 1991 recession more than does predictive ability of consumer confidence.

According to Batchelor and Dua, information about shocks that are non-economic in nature, like the 1991 Gulf War, are contained in consumer confidence measures. In these cases the ability of consumer confidence to predict expenditures would be increased.

The findings in Garner (1991) support this view. Garner compares forecast models with and without consumer confidence indices over times of “ordinary circumstances,” (late 1987 to the first half of 1990) and "exceptional circumstances," (late 1990 and 1991). Forecasts with consumer confidence performed worse during the ordinary times, but much better during the Gulf War time period. He suggests that the improved forecast ability of the models with consumer confidence is due to the fact that the Gulf War was an unanticipated, non-economic event.

This line of reasoning logically leads to the role of consumer confidence after a terrorist attack. Consumer confidence should be an important indicator of the economy following a terrorist attack; a shock to the economy of this sort leaves individuals little basis for forming expectations and making decisions using more traditional economic indicators. This view is not, however, borne out by the evidence following 9/11. Garner (2002) finds that the fall in consumer confidence (and the worsening recession) was not a result of these exceptional circumstances. Rather, the consumer confidence measures likely reflected the available economic information.

Garner's apparent contradiction of his 1991 findings points the way for further research into the role (if any) of consumer confidence in the economy. There is clearly a relationship between consumer confidence, as currently measured, and other economic indicators. The nature of that relationship is yet undiscovered. In particular, if consumer confidence does in fact contain important information in the wake of non-economic shocks, then it remains to be answered why consumers were so resilient after 9/11.

2.5 Implications

In a simplified model of the economy, individuals and firms respond to changes in prices by altering consumption and supply decisions based on maximizing individual utility or profit. This model abstracts from reality in that it gives little role to the decision-making process and has the implication that agents behave rationally. If it is acknowledged that agents often make decisions based on little or faulty information, or that the future is uncertain, then the model poorly explains economic outcomes.

Introducing uncertain outcomes into an economic model necessitates the consideration of how individuals perceive risk. By looking at individual choices as choices between different risky situations, decisions can be characterized as satisfying agents' desire to seek or avoid risk. Uncertainty also necessitates an understanding of expectations. Whether considered subjectively or objectively, individual expectations of event probabilities ultimately determine choices between different risky situations.

Despite what economists know about risk and uncertainty, its meaning for modeling responses to large shocks or terrorist attacks is, in a word, uncertain. From the consumer point of view (and the same argument could be made from the producer side), a likely scenario is that a terrorist attack increases the perceived probability of future economic hardship, and consumers respond by reducing current consumption. But consumer response to a terrorist attack depends on agents' perceptions of how event probabilities change, the individual level of risk aversion, and how expectations are formed; thus, as recent history suggests, a large shock will not necessarily lead to a large economic impact. Uncovering the conditions for such an effect is the direction for future research in this field.

3 Foundational Exploratory Simulations

This section introduces foundational simulations developed to explore the role of fundamental economic drivers in an agent-based framework. Specifically, these simulations explore the participation of firms and households in labor, goods, and financial markets.

3.1 Leisure Effects and Employment

Labor markets affect all other economic activity, including consumption and savings. Here, we present foundational simulations and findings focusing on labor markets and related factors, such as the demand for leisure and real-wage effects on the household's decision to work.

In these models, households earn income and consume goods to maximize utility. The *reservation wage* required to entice each household to enter the labor market varies across households, and the productivity rate of firms is constant across firms. Under this framework, our model allows us to infer the following logic: Some households produce more utility by not working for a firm if their *direct* personal utility productivity is greater than the utility that is generated *indirectly* by going to work, getting income, purchasing goods from firms, and then consuming those goods.[1] Full employment is defined as the level at which those who are willing and able to work for firms are employed. Actual employment is a combined function of households' willingness to work (which collectively defines the labor supply curve) and firms' productivity rates (which provide the production feasibility curve and related labor demand curve). Supply and demand in the goods market are functions not only of the price of goods in the market but also of the amount of labor that households are willing to supply to firms at the current wage rates and goods prices.

3.1.1 Households

Each household is a single potential laborer endowed with its own internal productivity, ρ_h, which represents the quantity of goods the household can produce for its own consumption each time period if it chooses *not* to work for a firm. The population of households has internal productivity rates uniformly distributed over the interval $[\rho_{min}, \rho_{max}]$.

Each household tries to maximize its utility each time period by consuming either home-produced goods or firm-produced goods. If consuming at home, its product is ρ_h. If consuming purchased goods, it will have earned a wage, w, from the firm and purchased q goods at price p in the market. Since its income-constraint equation is $p \times q_h \leq w$, a

[1] This is related to the notion of a household's trade-off between work and leisure. Leisure can be considered as a form of consumption for the household; that is, more leisure makes the household better off. However, spending time at work rather than at leisure generates income, which allows the household to consume goods and services. Households that can generate more utility from leisure will stay at home, while those that generate more utility from consumption will go to work. As described in Section 3.1.2, we model that switching point as a function of the ratio of wages to the price of produced goods.

household that works will be able to purchase and consume $q_h = w/p$ goods. The household's problem is then to select the consumption that solves

$$\max \mathrm{U}_h(\cdot) = \begin{Bmatrix} \frac{w}{p}, & \frac{w}{p} \geq \rho_h \\ \rho_h, & \frac{w}{p} < \rho_h \end{Bmatrix} = \max(\frac{w}{p}, \rho_h). \tag{2}$$

3.1.2 Labor Supply

Labor supply is defined by the number of households willing and able to work for firms. Given the uniform density function of internal productivities, $[\rho_{min}, \rho_{max}]$, the labor supply can be defined as

$$L^s = L^s(\frac{w}{p}) = \int_{\rho_{min}}^{\rho_{max}} f_h(\frac{w}{p})\ d\rho, \tag{3}$$

where

$$f_h(\frac{w}{p}) = \begin{Bmatrix} \frac{H}{(\rho_{max} - \rho_{min})}, & \frac{w}{p} \geq \rho_h \\ 0, & \frac{w}{p} < \rho_h \end{Bmatrix}.$$

Substituting,

$$\begin{aligned} L^s(\frac{w}{p}) &= \int_{\rho_{min}}^{\frac{w}{p}} \frac{H}{(\rho_{max} - \rho_{min})} d\rho + \int_{\frac{w}{p}}^{\rho_{\max}} 0\ d\rho \\ &= H \left[\frac{\frac{w}{p} - \rho_{min}}{\rho_{max} - \rho_{min}} \right]. \end{aligned} \tag{4}$$

Figure 1 illustrates this supply function.

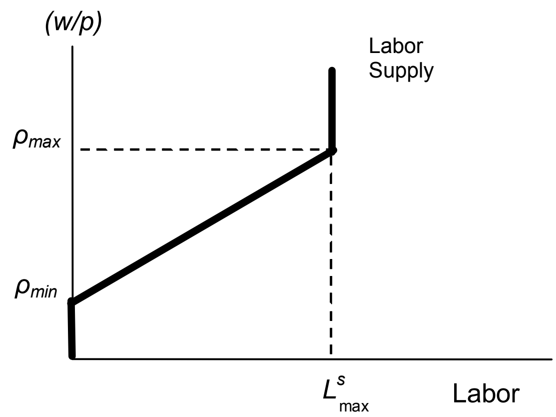


Figure 1. Labor supply function.

Full employment is defined as the condition where all households for which $\rho_h < (w/p)$ are employed by firms and the remaining are not employed by firms.

3.1.3 Goods Demand

Since each household maximizes utility by maximizing consumption, each household will purchase all it can, i.e., w/p. Goods demand follows as

$$
\begin{aligned}
G^D = G^D(\frac{w}{p}) &= \sum_{h=0}^{H} q_h \\
&= \left(\frac{w}{p}\right) L^s(\frac{w}{p}) \\
&= \left(\frac{w}{p}\right) \mathrm{H} \left[\frac{\frac{w}{p} - \rho_{min}}{\rho_{max} - \rho_{min}} \right].
\end{aligned} \tag{5}
$$

Households select a firm in each period from which to purchase based on a quasi-rational discrete choice function. Formally, a firm with price p_f has a purchase probability of

$$
\text{Prob[household selecting firm } f\text{]} = \frac{{p_f}^{\gamma}}{\sum_{f=1}^{F} (p_f)^{\gamma}}, \tag{6}
$$

where $\gamma = -1$ in this exercise. This decision rule has the following implication: if a firm has a price that is one-half of another firm's price, then it has twice the probability of being selected by a household.

3.1.4 Firms

Each time period, firms use labor l_f to produce output, using the production technology

$$
q_f = q_f(\rho_f, l_f) = \rho_f \times l_f, \tag{7}
$$

where ρ_f represents the conversion of labor to goods.

Labor is purchased in the labor market at the fixed rate *w*. (We could also model *w* as varying, but this would add complexity to the equilibrium and market dynamics that is outside the scope of this work.) The produced goods, on the other hand, are sold in the goods market at price p_f as determined by the firm. The firm's problem is to select the price that maximizes profits, i.e.,

$$\begin{aligned} \max \pi_f &= p_f q_f - w l_f \\ &= p_f \rho_f l_f - w l_f \\ &= (p_f \rho_f - w) l_f . \end{aligned} \tag{8}$$

If we impose the condition that a firm will only produce if $\pi_f > 0$, then we must have the condition $(p_f \rho_f - w) > 0$, or $\rho_f > \frac{w}{p_f}$. If so, the firm will try to produce infinite profits by hiring infinite labor. Summarizing, optimal firm production is then

$$\hat{q}_f = q_f(\frac{w}{p}) = \left\{ \begin{array}{ll} +\infty, & \frac{w}{p_f} < \rho_f \\ 0, & \frac{w}{p_f} \geq \rho_f \end{array} \right\}, \tag{9}$$

the goods supply function in the goods market is

$$G^S = \sum_{f=1}^{F} \hat{q}_f , \tag{10}$$

and the labor demand function in the labor market is

$$L^D = \sum_{f=1}^{F} \hat{l}_f . \tag{11}$$

3.1.5 Market Equilibrium

For market clearing, we have the necessary conditions $G^D = G^S$ and $L^D = L^S$. We consider two specific cases: a single-firm monopoly economy and a multiple-firm competitive-market economy. To provide a reference point, we also compute some properties for the socially optimal economy, where firms have zero expected profits and all laborers that are willing and able to work are employed by firms.

3.1.6 Simulation Parameters and Mechanics

To investigate whether the economy "discovers" its monopoly or competitive equilibrium and, if so, how this process occurs, we conducted a series of Aspen[2] agent simulations for the single-firm (monopoly) and multiple-firm (oligopoly) cases. In both cases we used the socially optimal economy as comparative, limit conditions. The parameters for the two cases are given in Table 2.

Table 2. Simulation Parameters

Parameter		Case 1 Monopoly	Case 2 Monopolistically Competitive	Socially Optimal Competition
Households				
	Number	100	100	---
	Internal productivity	[1.0, 2.5]	[1.0, 2.5]	[1.0, 2.5]
Firms				
	Number	1	5	8+
	Productivity	2.0	2.0	2.0
	Wage rate offered	50.0	50.0	50.0
Calculated Optima				
	Equilibrium price ($\hat{p}$)	70.7	31.6	25.0
	Equilibrium employment (L^s)	29	64	80

Each household monitors wages and prices every time step. If an unemployed household h with internal productivity ρ_h observes that $(w/p) > \rho_h$, then that household applies for employment. If an employed household with internal productivity ρ_h observes that $(w/p) < \rho_h$, then that household quits its job and leaves the labor market. In this idealized example, each employed household attempts each time period to spend its entire wages on the purchase of goods for consumption.

Each firm employs an algorithm for setting the price of the goods that it sells in the goods market. This algorithm essentially searches for the price that maximizes current-period profits by averaging its price and profit performance over time. We chose this algorithm for its ability to converge for this particular exercise. (Typically, Aspen employs more complex learning algorithms, e.g., Slepoy and Pryor 2002.)

Each firm also runs a simple algorithm to determine the production that will maximize its profits, and then tries to access the labor necessary to produce that amount of output. This algorithm continues to scale its labor force in a certain direction (up or down) so long as the firm's profits are increasing.

If the firm reaches a local maximum, it oscillates about that number of employees while perpetually searching nearby for an employment level that will increase its profits. Finally, each firm purchases any excess inventory of goods that it was unable to sell in the goods market.

[2] For details on the structure and uses of the Aspen model, see Basu et al. 1996 and Basu and Pryor 1997.

3.1.7 Simulation Results: Monopoly versus Oligopoly

In the following graphs, green represents this competitive case, and red represents the previous monopoly case. In the competitive case, we find that firms collectively employ more labor than in the monopoly case. The number of employees converges near (slightly above) the expected competitive employment level of 64 (Figure 2), and the price converges slightly below the competitive price of $31.60 (Figure 3).

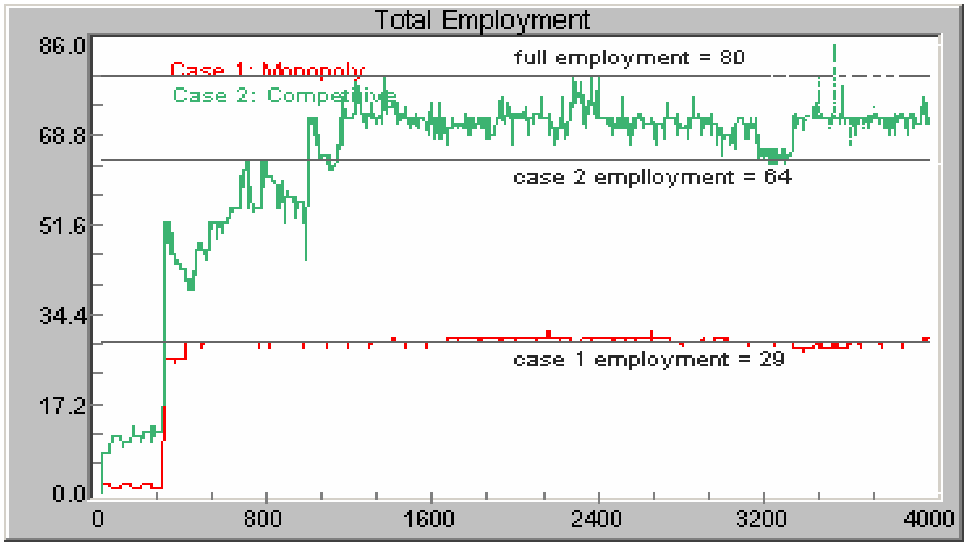


Figure 2. Competitive case: Total employment.

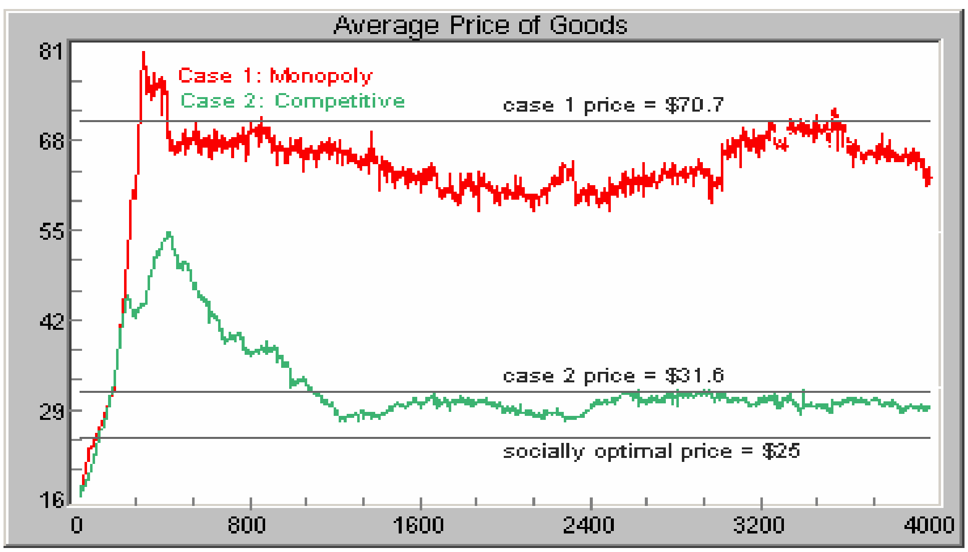


Figure 3. Competitive case: Average price.

Comparing the two cases, we find in the competitive case that firms collectively generate lower profits (about half) than in the monopoly case (Figure 4) and that they also employ more laborers and provide higher real wages to the households than in the monopoly case (Figure 5).

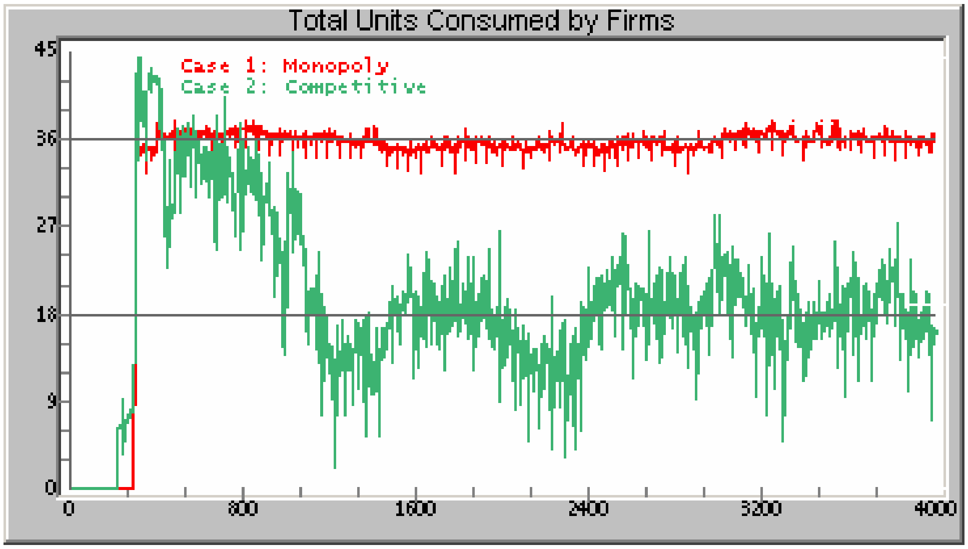


Figure 4. Competitive case: Firm profits.

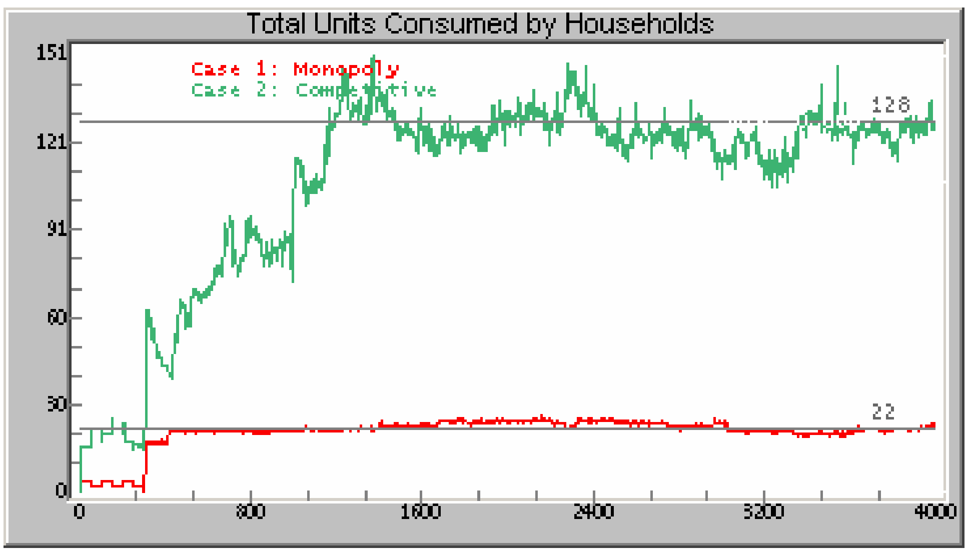


Figure 5. Competitive case: Units consumed.

We also find that under competition each household's real wage is more than double the wage earned under the monopoly case (Figure 12). In the competitive case, households retain 1.75 units per time step (87% of marginal product), compared with 0.75 units (38% of marginal product) in the monopoly case.

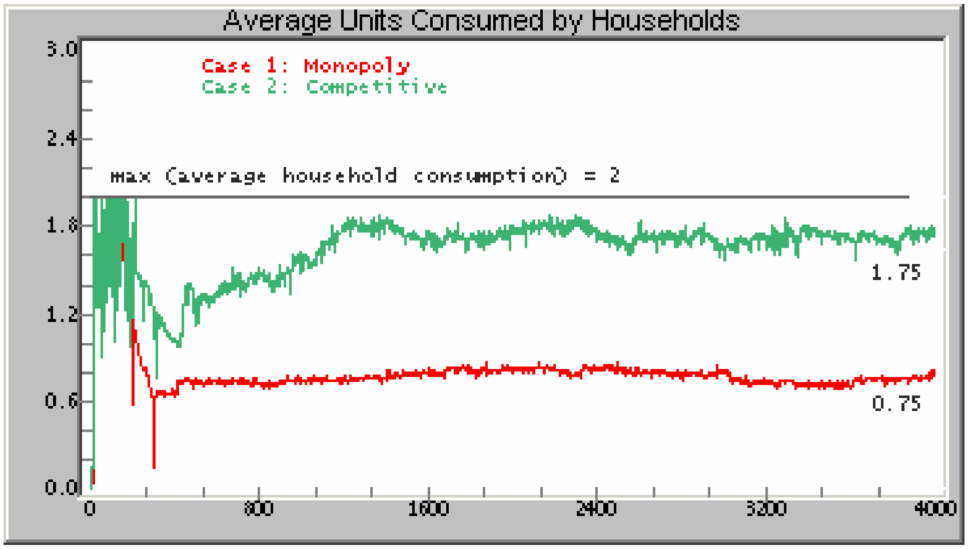


Figure 6. Competitive case: Average units consumed.

3.1.8 Discussion

In this economy, a fundamental role of the firm(s) is to allow households with lower-than-average personal productivities to contribute to, and get returns from, higher-productivity activities offered by firms. Per capita income increases for these lower-than-average households. Increasing the number of firms will increase the returns to households; output and employment increases, prices decrease, and households buy more goods.

In the single-firm economy, the firm gets fairly accurate short-run, if not long-run, information about the number of goods it can sell and about the available labor force. In contrast, as seen in the multiple-firm figures, the multiple-firm economy can have significant "noise" in both markets. More firms are asynchronously assuming their employment is fixed; employment shifts in and out of firms and *across* firms, making employment response difficult to interpret. Compare the total employment levels displayed previously in Figure 2: while monopoly employment quickly approaches equilibrium, the competitive employment fluctuates significantly over almost half of the time steps.

Similarly, multiple firms are trying to experiment on price and reacting to the perceived demand response to their price. If the noise created by experimentation is high, the signal-to-noise ratio will be low, making convergence to equilibrium more difficult.

Similarly, in the multiple-firm economy, as each firm is individually searching for an optimal price, the ever-changing set of prices from the *other* firms creates noise in the first firm's interpretation of the effect of its price change on its demand. This uncertainty propagates back through production, employment, and ultimately its demand again.

In this model, the firms have a memory of several time steps and use moving averages of several time steps to estimate the success of their pricing and employment choices. We also experimented with very shallow memories in which the firms looked only at the last time period. In those simulations, we often found that the firms would repeatedly overcompensate and that the markets would diverge into unexpected corner solutions. We found that simple smoothing avoids some of the divergence issues encountered in other discrete simulations (e.g., Arifovic 1994), leads to more reasonable and robust results, and probably provides a more realistic firm behavior.

Firms most directly experience the bottom-line impact of noise in their profits, which we observe in Figure 4. We can see that the monopoly profits quickly converge to equilibrium, whereas the competitive profits suffer from both short-run noise and long-run oscillations. We illustrate this contrast more precisely in Figure 7, which plots the average of the firms' standard deviation of profit (from Figure 10) for the 50 most recent time steps. We see that the average standard deviation of profit is on average seven fold higher in a competitive market than in a monopoly market.

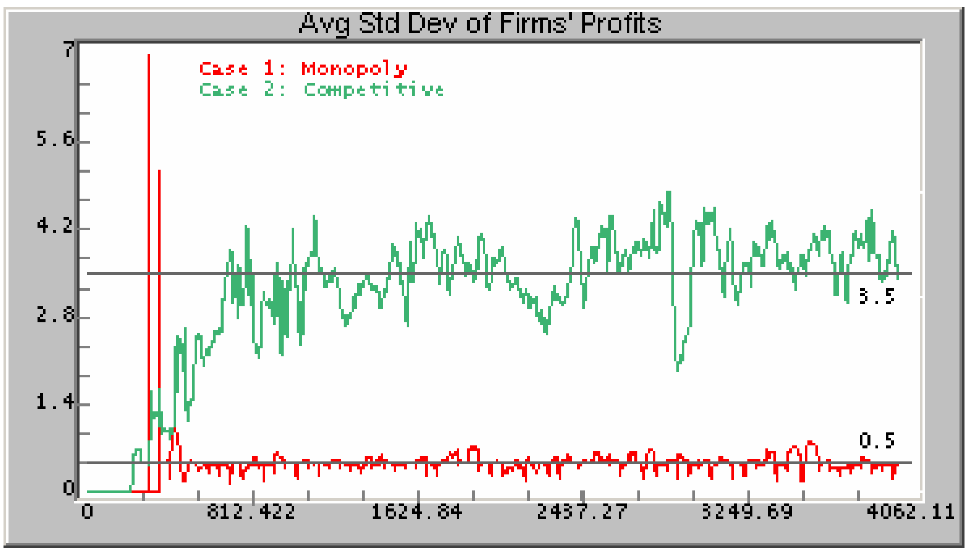


Figure 7. Variability in firms' profits: Monopoly and competitive cases.

While a terrorist attack arguably has direct impacts on households' willingness to consume particular goods (or goods at all), such an attack could also create noise in markets that could easily be interpreted as economic instability or downturn. Consider an attack where a significant fraction of firms in an industry were disabled for a period of time. Endogenously, as implied by this model, output and employment could decrease and prices could increase sharply, as the remaining firms have a clearer picture of their customer-response functions. Furthermore, as the disabled firms return and rush to regain market share, the ensuing noise in prices, output, and employment could be interpreted by still-uneasy consumers as a floundering economy, thereby perpetuating a lack of confidence and poor performance in the embattled sector. The mechanics of such perpetuating perceptions needs to be better understood.

The comparison of the two sets of simulations also illustrates how competitive markets can, given constant demand, be "self-healing": if during a disruption several firms are incapacitated, the remaining firms can use the new-found market power to increase profits, get clearer information on (an increased) demand, and stabilize until the incapacitated firms return. From this model, however, returning firms imply increased noise in goods and labor markets, creating instabilities.

In general, this work demonstrates the nature of competition and full employment in a closed economy in which income effects are pervasive and the goods' supply curve is explicitly tied to work effort. By capturing these output and employment features of the macroeconomy, we have demonstrated the appropriateness of agent-based models for microbased macroeconomic analysis, particularly the ability to define computable equilibria and analyze the conditions under which these equilibria are stable within "small" fluctuations.

3.2 Life-Cycle Implications for Disruption and Recovery

This exercise introduces a bond market and savings into our agent-based model. Our goal in this paper is to demonstrate that labor, product, and bond markets converge to calculated equilibriums in accordance with a life-cycle hypothesis (LCH). We conduct an event study of an output disruption within the simulation, and find that the disruption cascades through all sectors affecting employment, consumption, and savings. Following the event, the simulation re-converges, with all markets returning to equilibrium at about the same time.

3.2.1 Life-Cycle Hypothesis

The life-cycle hypothesis (LCH) model (see Friedman 1957, Modigliani and Brunberg 1958, Ando and Modigliani 1963) defines household behavior as an attempt to smooth consumption patterns over one's lifetime somewhat independent of current levels of income; households do this by borrowing, saving, and lending. The model is typically represented as the following constrained maximization problem, as was previously presented in section 2.2:

$$\begin{gathered} \max U_{t=0} = \sum_{t=0}^{T(t)} \delta(U_t(C_t)) \\ s.t. \sum_{t=0}^{T(t)} \rho(C_t) = B_{t=0} + \sum_{t=0}^{L(t)} \rho(Y_t), \end{gathered} \tag{12}$$

where C_t and Y_t are respectively the levels of consumption and income in period t, $U_t(C_t)$ is the *utility* received from consumption in time period t, B_0 is an initial wealth endowment, and $\delta(\cdot)$ and $\rho(\cdot)$ are discount functions; $\delta(\cdot)$ discounts the value of future utility according to the household's internal time preference, and $\rho(\cdot)$ discounts the value of future consumption and income according to the market interest rate. Also, $L(t) \leq T(t)$, where $L(t)$ denotes the number of remaining periods in which the household can work in the labor market to earn income, and $T(t)$ denotes the number of remaining periods in the household's life cycle.

The model used in this paper includes several simplifying assumptions. Specifically, each household's utility function is constant across time:

$$U_t(C_t) = U_\tau(C_\tau) = U(C), \qquad (13)$$

utility is a concave function of consumption:

$$U_t^{'}(C_t) > 0 \text{ and } U_t^{''}(C_t) < 0, \qquad (14)$$

households do not discount the value of future consumption:

$$\delta(C_t) \equiv C_t, \qquad (15)$$

wages are constant across firms and across time:

$$Y_t = Y, \qquad (16)$$

and the market interest rate is zero:

$$\rho(C_t) \equiv C_t \text{ and } \rho(Y_t) \equiv Y_t. \qquad (17)$$

The assumptions of equation (2) allow us to simplify the household's optimization problem as follows:

$$\max U_{t=0} = \sum_{t=0}^{T(t)} C_t = B_{t=0} + (L(t) \times Y) \qquad (18)$$

By inspection of equation (2b), it follows that households will seek to balance their consumption equally across time periods as follows:

$$\overline{C}_t = (\, B_t + (L(t) \times Y)\,) \,/\, T(t) \qquad (19)$$

3.2.2 Savings, Retirement, and the Bond Market

Households of age A for which $L > 0$ are said to be *career* households, and are able to work in the labor market to earn income. Households of age A such that $L = 0$ and $T > 0$ are said to be *retired* households, and are unable to work. Continuing from above, career households will seek to maintain a constant savings rate, as follows:

$$\overline{S}_t = Y - \overline{C}_t \qquad (20)$$

In this paper, bonds are strictly a mechanism for interest-free intertemporal substitution between generations of households.[3] We model the bond market as a continuous double auction conducted by a bond exchange that takes limit orders from households. Each order includes a buy/sell indicator and the number of shares to be traded. For the purposes of this paper, bond prices are fixed at $1. The exchange will therefore clear a volume equal to the minimum volume of open buy orders or open sell

[3] For bonds as a government-issued asset class in Aspen, see Basu et al. 1998.

orders. For example, if there are more open buys than sells, then the exchange will clear all open sells and the remaining buys will carry over to the next time step.

The aggregate number of bonds in the economy remains constant throughout the simulation. Career households accrue wealth *(B)* in the form of bond holdings via periodic saving contributions, such that

$$B_t = B_{t-1} + S_t . \tag{21}$$

Retired households cannot accrue bonds, but rather consume by selling bonds in order to purchase goods, such that

$$B_t = B_{t-1} - C_t . \tag{22}$$

The bond market is facilitated by a bond exchange agent, which simply clears the queue of open limit orders from households. Specifically, the exchange reads and sorts all limit orders according to limit price, then matches buys to sells in such order to maximize the trading volume, and finally sends transaction notices to all households whose orders were executed. The bond exchange executes partial-fill transactions when the size of buy and sell orders are unequal.

3.2.3 Market Equilibrium

Bond market equilibrium is linked to equilibrium in the other markets. For example, if the economy experiences a period of recessionary unemployment, then a portion of career households will be unable to maintain their planned rate of consumption and bond purchases. Under conventional wisdom, one would expect the average household to both reduce consumption and fall behind on bond purchases, leading to a surplus in the bonds market. Furthermore, if households face a finite time horizon, then temporarily unemployed households will reduce their expected average consumption for the remainder of their life cycles, and therefore reduce their target bond holdings. We should note that if we allowed bond prices to adjust, then retired households would face falling bonds prices after a recession, leading to a reduction in both wealth and expected total consumption for the remainder of their life cycles.

3.2.4 Simulation Parameters and Mechanics

We explore convergence in a short-run model under the assumption of infinite time horizons for household decisions (model 1) and long-run model under the assumption of finite time horizons for household decisions (model 2).

3.2.4.1 Infinite Horizons

Households in model 1 are instantiated with a initial age, which does not change as the simulation runs. That is, households do not grow older or approach their life-cycle horizons. This scenario is synonomous to a life-cycle model with an infinite time horizon, which could be used to simulate a short timespan relative to the lifespan of the household.

At the start of the simulation, such households are assigned a fixed target level of bondholdings (i.e. target amount of savings); this target remains constant throughout the simulation. Each household is also given an endowment of bonds, the size of which is randomly drawn from a uniform interval centered about the assigned target bondholding. Thus, each household's endowment is either greater than, equal to, or less than its assigned target bondholdings, within a fixed variance.

Throughout the simulation, households with excess bonds submit orders to sell their excess bonds on the exchange. Households that are deficient in bonds submit orders to buy bonds as they earn income. Each household submits orders until it eventually achieves its target level of bondholdings. Once either all sellers or all buyers achieve their target bondholdings, the market is said to have cleared, and no further bond exchanges will occur.

3.2.4.2 Finite Horizons

Model 2 assumes overlapping generations in which each household ages and saves according to equations (19) and (20). At the start of the simulation, each household is randomly assigned an age $A_{t=0}$ from a continuous uniform interval from 20 to 80. Retirement occurs at age 60 for all households. The following algorithm assigns a target level of bondholdings to each household based on its initial age, retirement age, and wage rate:

1. Compute expected average consumption per period ($\overline{C}_t$) based on equations (3) and (4) assuming $B_{t=0}$ = 0 and $L(0) = max(L)$ = 40.

2. Compute the expected average savings contribution per period ($\overline{S}_t$) based on equation (5) and $\overline{C}_t$ from above.

3. If the household is initially of career age $A_{t=0}$<60, then compute the initial target bondholdings ($B_{t=0}$) as follows:

$$B_{t=0} = (A_{t=0} - 20) \times \overline{S}_t \times (\text{time steps per year})\,. \tag{23}$$

Otherwise, if $A_{t=0}$> 60, then compute the initial target bondholdings as follows:

$$B_{t=0} = B_{Age=60} - ((A_{t=0} - 60) \times \overline{C}_t \times (\text{time steps per year})). \tag{24}$$

The initial *target* bondholdings increase across career households when sorted by age from 20 to 60, and decrease across retired households when sorted by age from 60 to 80.

As was also the case for bond-market households, at the start of the simulation, each life-cycle household is given an endowment of bonds drawn from a uniform interval.

Throughout the simulation, households grow older with each time step and revise their target bondholdings based on their new age and current bondholdings. Ideally, a household will purchase bonds at a constant rate from age 20 to 60, leave the workforce at age 60, and sell bonds from age 60 to 80 using the proceeds for consumption. Once a household expires at age 80, it is recycled as a new household (heir) of age 20; any bonds held by a household upon expiration are endowed to its heir as a type of accidental bequest.

To demonstrate that labor, goods, and bond markets simultaneously converge to calculated equilibria under both finite and infinite time horizons, we introduce new household parameters for modeling life cycles: *age* and *time-steps-per-year*. The *equilibrium bond holdings* is a constant in model 1, but varies in model 2 with respect to age according to equation (23). The *deviation in bonds* refers to the average percent deviation between actual and target bond holdings across households. In model 1, the equilibrium deviation *(Δ)* is constant based on the net difference between initial bond deficits and surpluses. In model 2, the equilibrium deviation *(Δ(t))* will fluctuate over time as a function of household characteristics.

Table 3. Simulation Parameters

Parameter		Model 1 Infinite Horizon	Model 2 Finite Horizon
Households			
	Number	100	100
	Internal productivity	[1.0, 2.5]	[1.0, 2.0)
	Age	----	[20, 80]
	Time steps per year	---	100
Firms			
	Number	5	5
	Productivity	2.0	2.0
	Wage rage offered	50.0	50.0
Calculated Optima			
	Equilibrium price ($\hat{p}$)	31.6	31.6
	Equilibrium employment (L^s)	59	59
	Equilibrium bond holdings	200	f(age)
	Equilibrium deviation in bonds	Δ	Δ(t)

3.2.5 Simulation Results

The figures in this section display time-step plots of simulation variables. In each case, the horizontal axis represents time-step iterations.

3.2.5.1 Model 1: Infinite Horizons

To investigate the impact that the bond market has on labor and goods markets, We will compare the simulation results for two cases. Case 1 refers to the case of no bond market, or a *cleared bond market*.[4] Case 2 refers to a *converging bond market* in which we initiate the simulation with the bond market in disequilibrium. In case 2, households with a bond deficit must earn income to buy bonds; this process takes time. We track the market clearing process in Figure 8 by plotting the average percent deviation across households between their target and actual bond holdings. Since case 1 begins with a cleared bond market, the deviation for case 1 is always zero. Case 2 slowly clears until it reaches a minimum deviation at time step 2560; at which time the sellers' market has cleared, but a few buyers remain.[5]

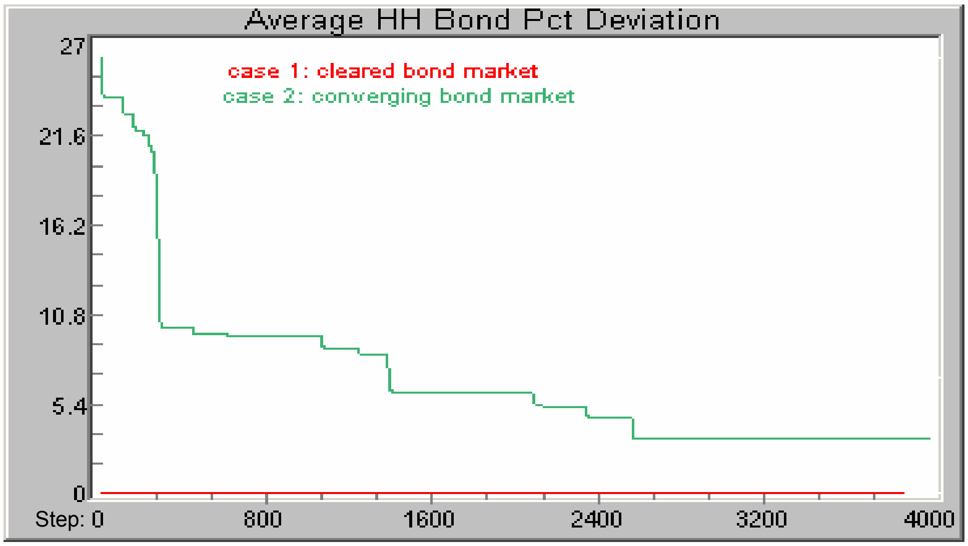


Figure 8. Infinite horizon: deviation in bond holdings.

Although the convergence process in the bond market for case 2 takes time, it does not significantly affect convergence in the other markets. For example, Figure 9 shows that the number of employees converges in both cases at roughly the same rate to slightly above the expected competitive employment level of 59. Similarly, the price converges in both cases at roughly the same rate to slightly below the competitive price of $31.60 (Figure 10).

---

[4] In the case of infinite horizons, once the bond market clears households have no further incentive to substitute between consumption and bond holdings; that is, *"once cleared, always cleared"*. Therefore, from the perspective of the labor and product markets, a *cleared* bonds market is synonymous with *no* bonds market at all. So, if we initialize all households such that actual and target bond holdings are equal, then other variables in the simulation will converge as if there was no bonds market in the simulated economy.

[5] There are two reasons that buy orders remain in queue at the bond exchange even after the bond market clears. First, the aggregate bond deficit did not equal the aggregate bond surplus at the start of the simulation. Second, this model does not allow for time preference and bond-price adjustments; these features will be addressed in subsequent models.

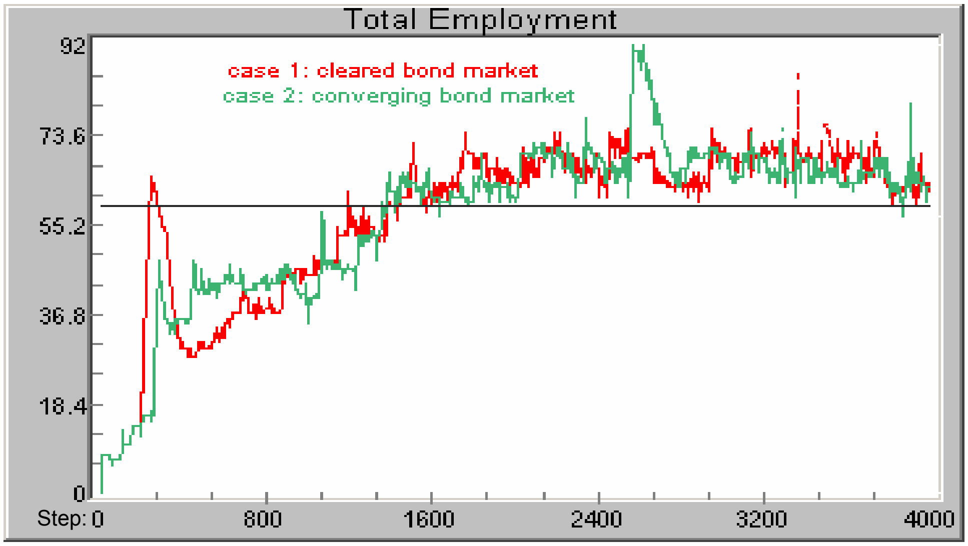


Figure 9. Infinite horizon: total employment.

Figure 10 also shows that firms searching for price in the face of a converging bond market (case 2) are more tightly constrained from raising price during the correction phase (first 1000 time steps) than when the bond market is already clear (case 2). This is true because firms in case 2 must compete against the households' demand for bonds.

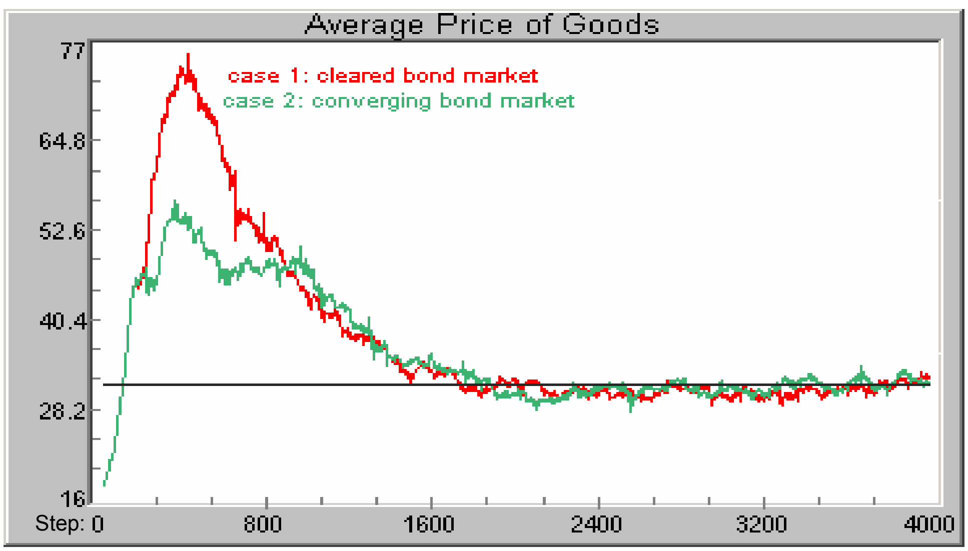


Figure 10. Infinite horizon: market price of goods.

Figure 11 and Figure 12 show that the presence of a converging bond market, compared to a cleared bond market, has no appreciable impact on firm profits or household consumption in this model.

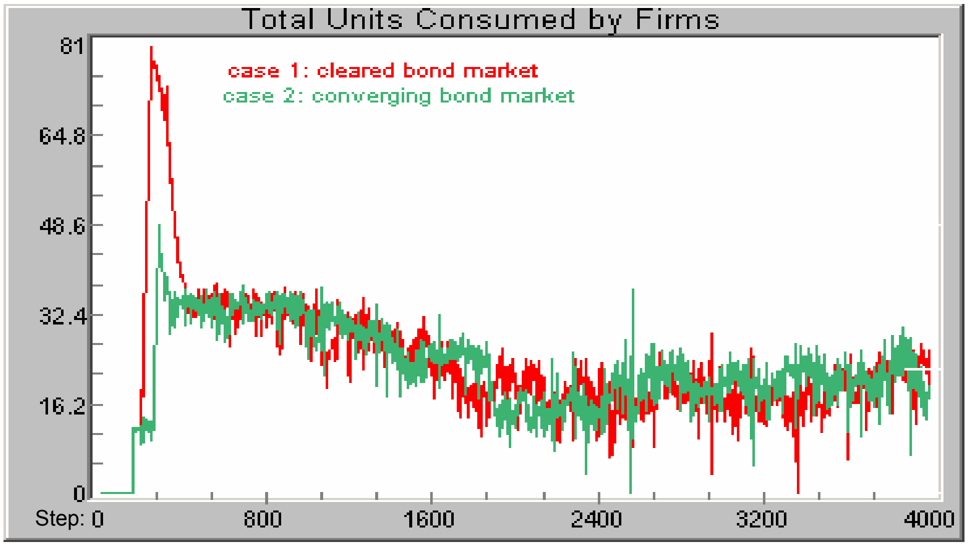


Figure 11. Infinite horizon: firm profits.

The spike in household consumption (Figure 12) at time step 2560 for case 2 occurs when the bond-sellers' market clears, thereby precluding further bond purchases and leading bond buyers to increase their consumption rate.

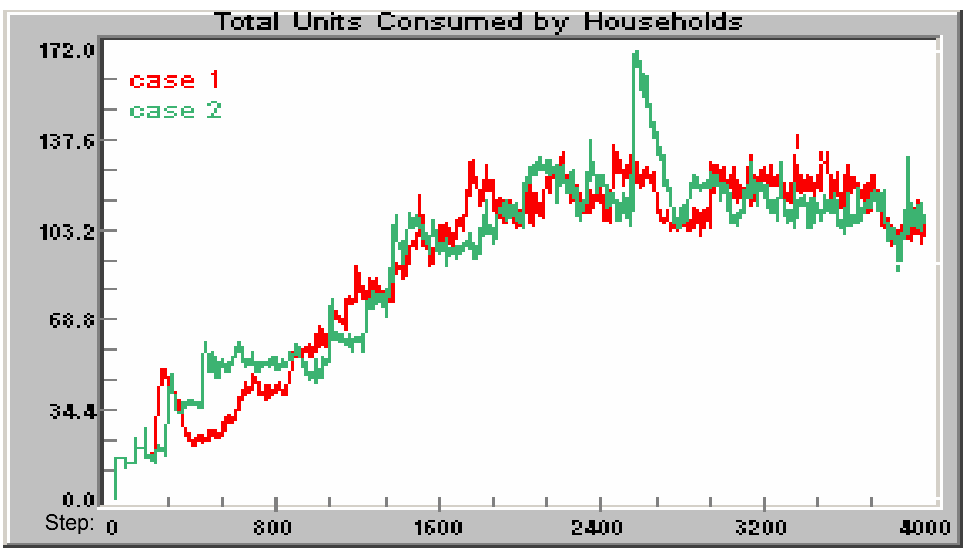


Figure 12. Infinite horizon: units consumed.

3.2.5.2 Model 2: Finite Horizon Households

The finite horizon model entails continuous overlapping generations of aging households. In this model, the equilibrium conditions are not fixed. For example, the equilibrium number of jobs under monopolistic competition is not only a function of the competitive price ($31.60), but also depends on the number of households in the career phase (i.e. age less than 60), which changes over time as households age and retire and expire. Figure 13 shows that the equilibrium number of jobs fluctuates over time near 49 jobs.

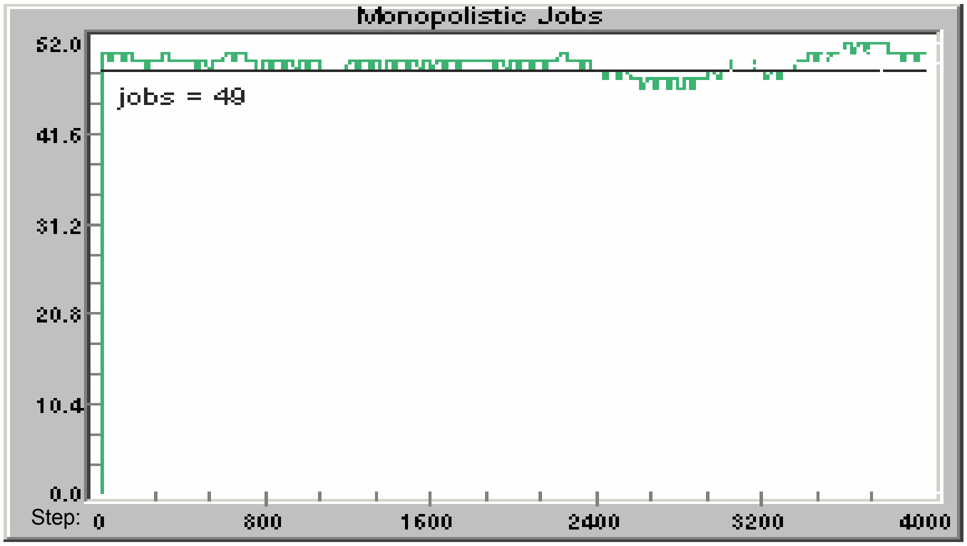


Figure 13. Finite horizon: equilibrium employment.

Nevertheless, we still find, as in previous simulations, that the number of employees converges to slightly above the expected competitive employment level of 49 (Figure (14). Similarly, the price converges to the competitive price of \$31.60 (Figure 15).

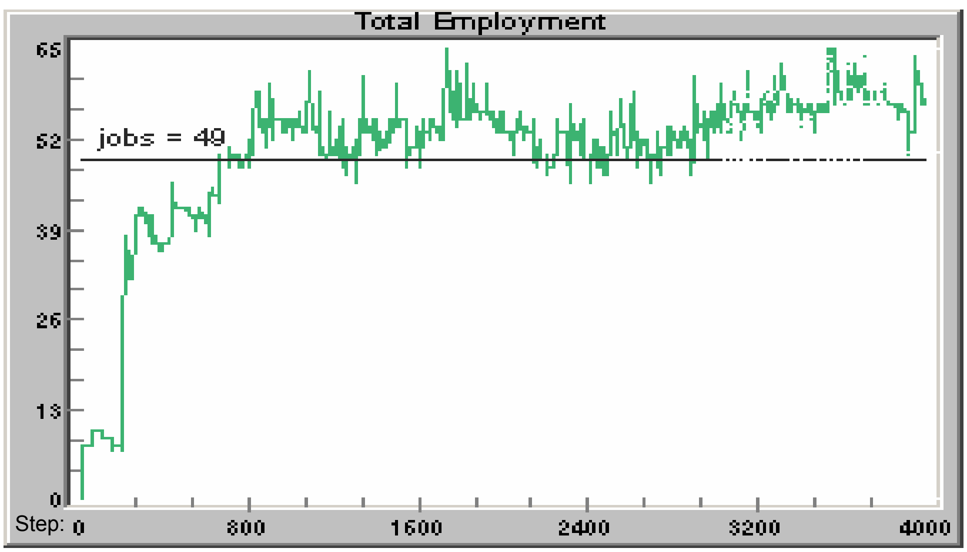


Figure 14. Finite horizon: total employment.

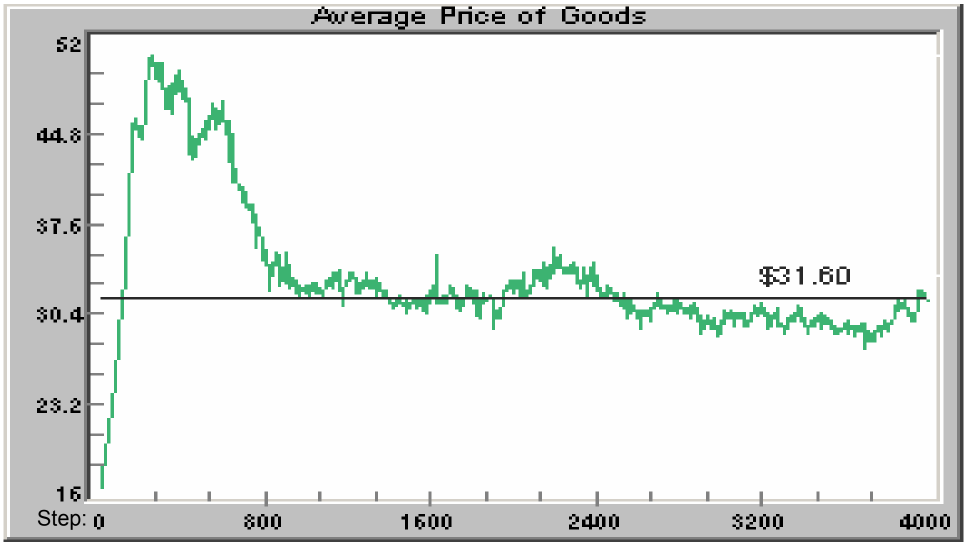


Figure 15. Finite horizon: market price of goods.

Under a finite horizon, because each household's target bond holdings are constantly changing, the average percent deviation across households between their target and actual bond holdings is not always decreasing, but rather fluctuates (see Figure 16), presumably in conjunction with the equilibrium deviation $\Delta(t)$ from Table 2.

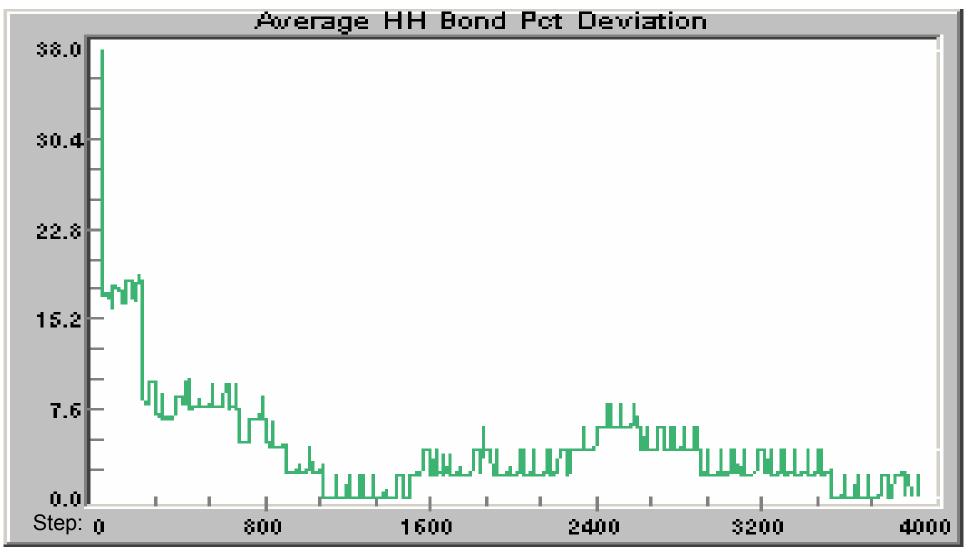


Figure 16. Finite horizon: deviation in bond holdings.

3.2.6 Event Study of an Output Disruption with Finite Horizons

Our underlying objective is to develop a method for estimating the expected economic impact of a terrorist event, such as an output disruption to a productive sector of the economy. To observe the effects of an output disruption, we ran two simulations assuming finite horizons. We ran a baseline simulation (case 1) with no disruption to show baseline economic activity, and an event simulation (case 2) to show the relative effects of a disruption. We find that an output disruption has cascading effects long after the event window, and that recovery in the bonds market corresponds to recovery in the labor and product markets.

The simulation begins in disequilibrium, but converges to equilibrium after the first 1000 time steps. We imposed an output disruption from time step 2001 through step 2100, during which firms were precluded from producing goods.

3.2.6.1 Impacts to Labor and Product Markets

We find that an output disruption occurring from time step 2001 to 2100 leads to substantial cascading effects in the labor and product markets that continue through time step 3800. Figure 17 shows that the event window is followed by an enduring period of unemployment. Figure 18 shows delayed effects on firm profits, and Figure 19 shows similar impacts to household consumption. All of these variables appear to re-converge to baseline around time step 3800.

Figure 17. Event study of total employment.

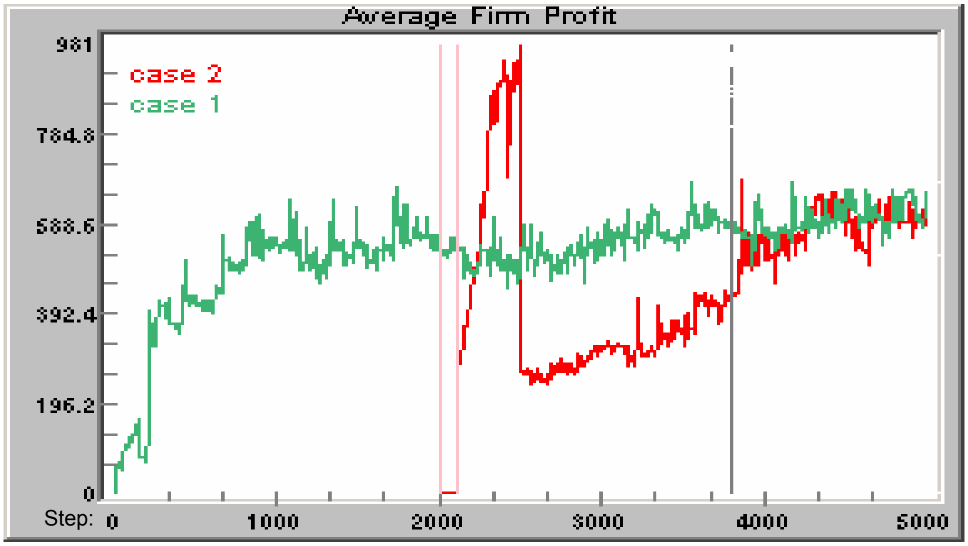


Figure 18. Event study of firm profits.

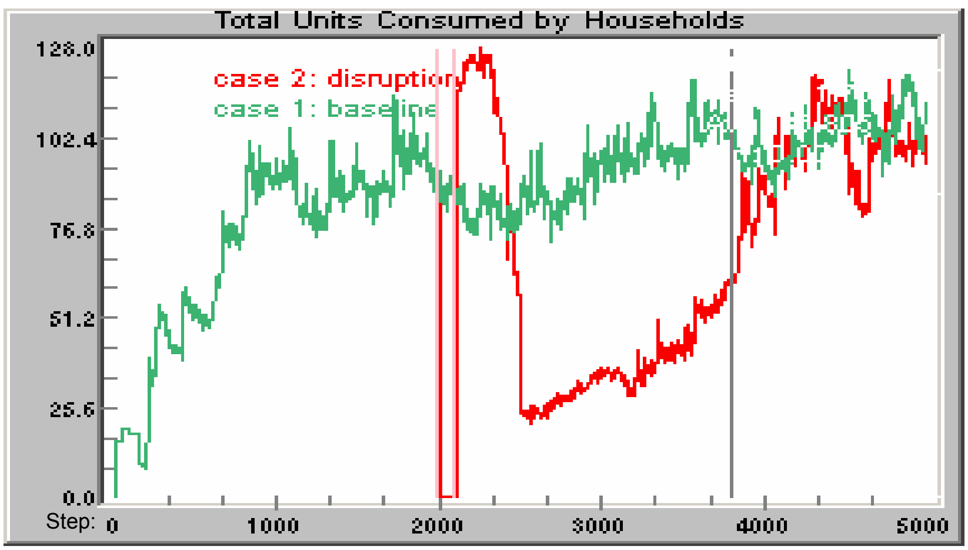


Figure 19. Event study of household consumption.

3.2.6.2 Impacts to Bond Market

The cascading effects from the disruption are also found in the bond market. Figure 20 shows a substantial deviation between target and actual bond holdings following disruption; these impacts enduring in the bond market for the same period as those in the labor and product markets: through time step 3800.

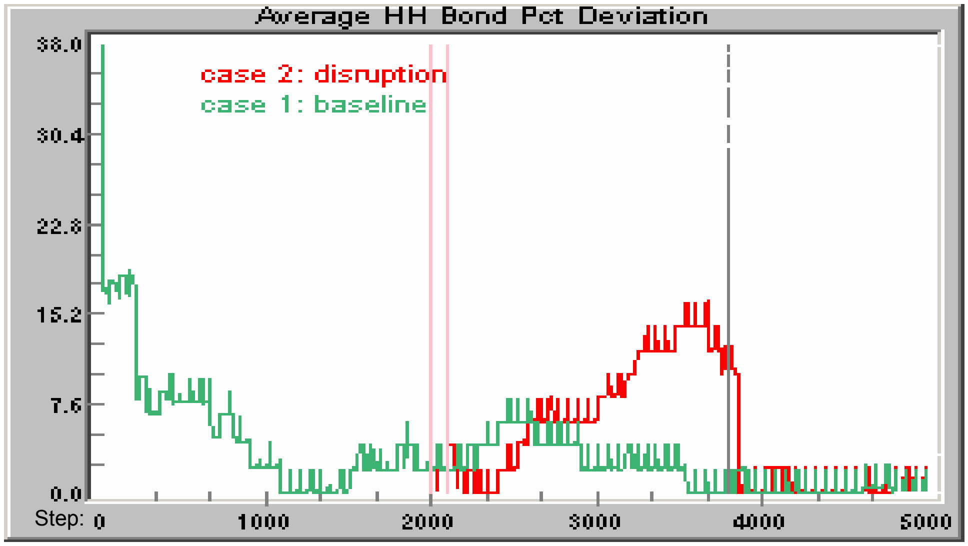


Figure 20. Event study of deviation in bond holdings.

Further inspection reveals the source of the deviation in bond holdings. The recessionary unemployment shown in Figure 17 implies that unemployed households are unable to contribute to savings, which shrinks the aggregate bonds budget (Figure 21).

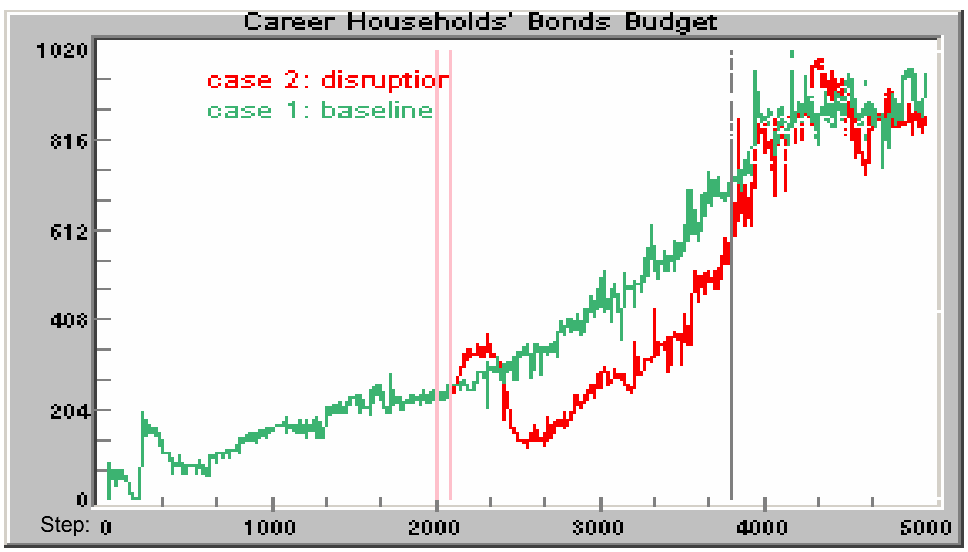


Figure 21. Event study of career households' budget for bonds.

Perhaps the most fundamental impact in the bond market is the substantial impact that recessionary unemployment has on expected lifetime income, and therefore on target bond holdings. Persistent unemployment leads households to downgrade their estimated lifetime income. Under the LCH, such downgrades reduce the unemployed households' expected retirement bundle, which reduces the present value of that bundle, which reduces the households' target bond holdings. Figure 22 shows the result of such downgrades. We see that as soon as employment returns to equilibrium near time step 3800, the target bond holdings quickly jump back to the baseline for target bond holdings.

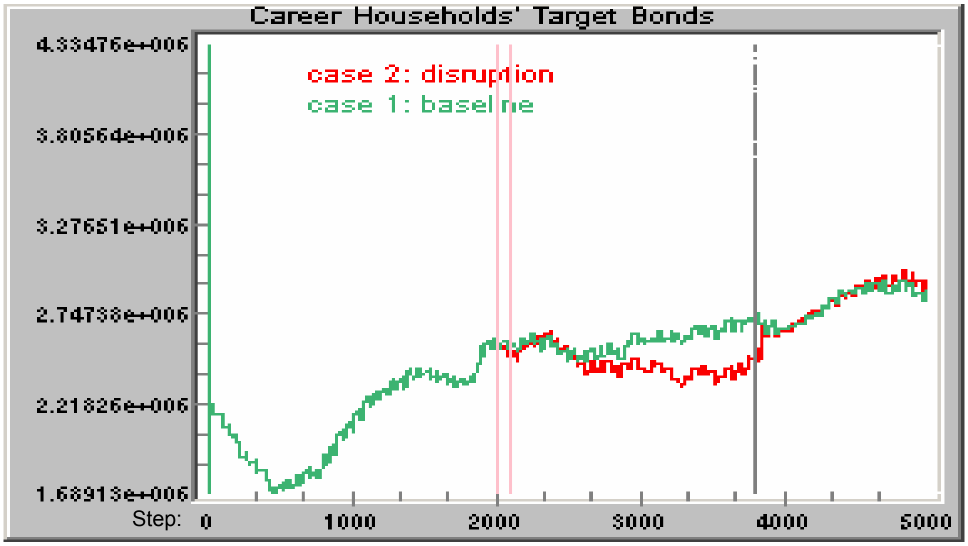


Figure 22. Event study of career households' target bond holdings.

This exercise demonstrates that agent-based simulations converge to calculated market equilibria in LCH models assuming infinite and finite horizons. It further models how a disruption to the productive sector can cascade through all markets, and that market recoveries are linked.

3.3 Remarks

These exercises help validate our computational approach and provide insight into the dynamic aspects of economic interactions. Many of these features are retained or extended in the model and simulation presented in the remainder of this report. One exception, however, is our choice of venue for financial interactions; to increase flexibility for borrowing and lending, and replace the bond market with a banking market. A key extension is the introduction of a more general set of life-cycle rules used for governing households' financial decisions.

4 The Analytic Model

The model builds upon the considerable foundation of life-cycle economics stemming from the early work of Fisher (1930), Friedman (1957), Modigliani and Brumberg (1958), and Ando and Modigliani (1963), and the overlapping-generations models of Samuelson (1958), Wallace (1980), Balasko et al. (1980-1981), and Tirole (1985). We model a discrete-time closed economy comprised of H households and F firms. Households decide how much to consume, borrow, and save each period. Firms decide whether to increase or decrease price and employment each period. Firms also act as passive lenders in a banking market by making their cash reserves available for loans to households. There is no money creation. For analytical convenience, wages, productivity rates, and interest rates are fixed, and marginal cost is constant and equal across firms.

Households grow older with each time period, and face a lifespan comprised of an employment-eligible (career) phase, during which households can earn wages in the labor market, and a retirement phase, during which households can only consume by withdrawing funds from their private savings. Households cannot substitute intertemporally by accumulating goods, but they can borrow funds from firms or deposit savings with firms via a banking market. Banking allows households to smooth their consumption patterns over their lifespans according to a conventional life-cycle hypothesis.

Each firm seeks to maximize short-run profit by hiring labor from households via the labor market, and producing goods to sell to households in the goods market. Firms earn *nominal* profits by charging prices above marginal cost and by charging interest on loans. Firms earn *real* profits by spending nominal profits to purchase back excess goods for their own consumption.

4.1 Household Consumption and Savings

We now derive the household's desired consumption expenditure and banking transaction in each time period. Consumption must be non-negative; a banking transaction can be either positive in the case of a deposit or negative in the case of a loan or withdraw.

Each household is comprised at any given time of a single individual who becomes employment-eligible at $Age_{\min}$, retires after Age_{retire}, and dies after $Age_{\max}$. Let Age_0 denote a household's current age measured in years, where $Age_{\min} \le Age_0 \le Age_{\max}$. Time is discrete, with a fixed number of λ periods per year. We define T_0 as the number of periods for consumption before the household expires, where

$$T_0 \equiv \lambda(Age_{\max} - Age_0) \begin{Bmatrix} >1 & \forall\ Age_0 < Age_{\max} \\ =1 & \forall\ Age_0 = Age_{\max} \end{Bmatrix}. \tag{25}$$

We define K_0 as the number of time-steps for earning income before the household retires, where

$$K_0 \equiv \lambda(Age_{retire} - Age_0) \begin{Bmatrix} > 1 & \forall\ Age_0 < Age_{retire} \\ = 1 & \forall\ Age_0 = Age_{retire} \\ = 0 & \forall\ Age_0 > Age_{retire} \end{Bmatrix}. \tag{26}$$

Any household that is employed by a firm supplies one unit of labor per period to its employer. All labor is supplied in discrete units, denoted as $l_t \in \{\text{positive integers}\}$.

Each household derives utility in period *t* by consuming q_t units of goods. Utility is defined as $u_t = (q_t)^\beta$, where β ≡ consumption elasticity ∈ (0, 1), so that $u_t' > 0$ and $u_t'' < 0$. Each household valuates future consumption with respect to its internal discount rate, d_h, which implies that the current utility derived from expected future consumption is

$$u_0 = \frac{u_t}{(1+d)^t} = \frac{(q_t)^\beta}{(1+d)^t}. \tag{27}$$

Households optimize the present-value of current and future utility by setting their consumption and savings rates according to the conventional life-cycle hypothesis, represented by the following constrained-maximization problem:

$$\begin{gathered} Maximize\ u_{t=0} = \sum_{t=0}^{T_0} \frac{u_t(q_t)}{(1+d)^t} \\ s.t.\ \sum_{t=0}^{T_0} \frac{c_t^e}{(1+r)^t} = b_{t=0} + \sum_{t=0}^{K_0} \frac{y_t^e}{(1+r)^t}, \end{gathered} \tag{28}$$

where b_0 denotes an initial wealth endowment, y_t^e is the expected nominal income earned in period t, and r denotes the market interest rate. Since there is no money creation, we employ a simplifying assumption that expected prices are equal across time, which implies $\frac{p_t^e}{p_0} = 1$. It can be shown that each household, in each period, will attempt to borrow or save to achieve current consumption of

$$\hat{c}_0 = p_0 \hat{q}_0 = \frac{b_0 + \sum_{t=0}^{K_0} \frac{y_t^e}{(1+r)^t}}{\sum_{t=0}^{T_0} \frac{(1+d)^{\frac{t}{\beta-1}}}{(1+r)^t}}. \tag{29}$$

The corresponding required savings transaction is

$$\hat{s}_0 = y_0 - \hat{c}_0, \tag{30}$$

where transactions are categorized as follows:

$$\begin{Bmatrix} deposit: & \hat{s}_0 \geq 0 & \forall b_0 \\ withdraw: & \hat{s}_0 < 0 & \text{and } b_0 > 0 \\ loan: & \hat{s}_0 < 0 & \text{and } b_0 \leq 0 \end{Bmatrix}. \tag{31}$$

We generalize the firm slection rule from equation (6) so that ϕ denotes the selection probability, and is defined as

$$\phi_{f_1} \equiv \Pr[\text{household } h \text{ selects firm } f_1] \equiv \frac{p_{f_1}^{\gamma}}{\sum_{f=0}^{F} p_f^{\gamma}}, \tag{32}$$

where $\gamma < -1$ is a constant, and p_f is the price charged by firm *f*. It can be shown that the relative probability of the household selecting firm f_1 over firm f_2 equals the scaled inverse of the firms' prices: $\frac{\phi_{f_1}}{\phi_{f_2}} = \left(\frac{p_{f_2}}{p_{f_1}}\right)^{|\gamma|}$.

4.2 Firms and Nash Equilibrium

For simplicity, wage rate *w* and productivity rate ρ in this exercise are fixed and equal across all firms. Firms search for the labor size and product price that maximize steady-state profits. A firm uses l_t units of labor to produce and supply q_t^S units of goods to the goods market using the production technology

$$q_t^S = \rho l_t. \tag{33}$$

Each firm sets its selling price p_t for goods, and earns production profit

$$\pi_t^{production} = \left(p_t q_t^{sold}\right) - wl_t \leq \left(p_t q_t^S\right) - wl_t = \left(p_t \rho - w\right) l_t. \tag{34}$$

Firms also participate as passive lenders in a banking market by making their cash reserves available for loans to households at the fixed market interest rate r. Interest earned from loans equals rA_t and provides a second source of profit to the firm. Summarizing, a firm's profit is defined by

$$\pi_t = \left(p_t q_t^{sold}\right) - wl_t + rA_t \,. \tag{35}$$

It can be shown (see Sprigg and Ehlen 2005) that Nash equilibrium price in the goods market is given by

$$q_f^D = q_f^S \quad \Leftrightarrow \quad p_f^* = \left[\frac{\rho \cdot g_t \cdot k_t \cdot \sigma_{f,t}}{w}\right]^{\frac{1}{\gamma-1}}, \tag{36}$$

where $\sigma_{f,t}$ denotes labor share, $k_t = \sum_{f=1}^{F} p_{f,t}^{\gamma}$, $g_t = \dfrac{1}{1-\theta_t}$, and $\theta_{S,t} \equiv \dfrac{S_t}{Y_t}$. That is, given its labor share $\sigma_{f,t}$ and the other firms' prices k_t, firm *f* cannot benefit by charging any price different from p_f^*.[6] The following partial derivative shows that equilibrium prices vary as the scaled inverse of labor share:

$$q_f^D = q_f^S \quad \Leftrightarrow \quad \frac{\partial p_f^*}{\partial \sigma_f} = \left(\tfrac{1}{\gamma-1}\right)\left[\frac{\rho \cdot g_t \cdot k_t}{w}\right]^{\frac{1}{\gamma-1}} \sigma^{\frac{\gamma}{1-\gamma}} < 0 \,. \tag{37}$$

Equations (36) and (37) imply that smaller firms are able to settle at higher prices; the reason is that smaller firms produce fewer goods and therefore require a smaller market share to maximize profit. This analytic result provides a baseline for validation of the simulation against economic theory.

5 Agent-Based Simulation

The prescribed model links the static equilibrium in the goods market to the dynamic equilibrium in the financial market, and derives the Nash equilibrium in the goods market as a function of consumers' sensitivity to price, represented by γ. A higher value for γ implies a household has a higher probability of selecting the firm with the lowest price

[6] To clarify, our model assumes that wages are fixed. Therefore, firms search for their optimal labor, but they cannot compete for labor by offering higher wages. This assumption introduces rigidity into the labor market, leading us to define Nash equilibrium in terms of goods prices alone. Although the firms are homogenous in all other respects, they are made heterogeneous by their relative allocations of labor. A more general model might relax this rigidity in the labor market, allowing one to define Nash equilibrium in terms of goods price and labor (p_f^*, l_f^*), in which case the firms would be truly homogenous and the Nash equilibrium would specify equal prices and labor shares.

(see equation (32)). If we were to assume that search costs are zero, then γ might be interpreted as the degree of rationality of quasi-rational consumers. This section describes a computer simulation of the analytical model in which the economic actors use a set of preferences, decisions, and processes that correspond to the analytical model, but differ in that firms in the simulation use quasi-rational adaptive search to search for their profit-maximizing behavior. The degree of rationality in this search is shown to be a pivotal factor in determining whether firms can collectively find and maintain the Nash equilibrium.

5.1 Simulation Mechanics and Parameters

5.1.1 Bank

A single bank agent serves as financial intermediary between households and firms. The bank holds the firms' excess reserves, which are made available as loans to households. The bank establishes an account for each household. Account balances can be positive or negative. Positive balances represent deposits, and accrue interest for the household at market interest rate *r*. Negative balances represent loans, and accrue interest for the bank at market interest rate *r*.

For simplicity, we assume that the bank applies the same interest rate to both loans and deposits, which implies that the bank, and therefore firms, should only engage in banking if households are net borrowers in the aggregate.

5.1.2 Households

At the start of each time period, each household makes two primary assessments. First, if an employment-eligible (career) household is unemployed, then it sends a job application to a firm. Second, all households calculate their target consumption and savings according to equations (29) and (30). These equations are calculated in part using income in the current period, which is known:

$$y_{h,t=0} = \left\{ \begin{array}{ll} 0, & \text{if unemployed} \\ w, & \text{if employed} \end{array} \right\}. \tag{38}$$

However, the calculation also requires career households to make assumptions regarding future income. In this simulation, future income is derived from employment history as follows. Each household keeps a record of its employment history, and calculates the average number of periods employed to estimate the probability of being employed in any future time period:

$$\Pr[\text{employed in future period } t] \equiv \sum_{t=1}^{N_h} \frac{y_{-t}}{w}, \tag{39}$$

where N_h denotes the number of periods retained in the household's memory. The household then probabilistically projects its income in each future period based on equation (39) as follows:

$$y^e_{t>0} = \begin{Bmatrix} w, & \text{with Pr[employed in } t] \\ 0, & \text{with Pr[}\textit{un}\text{employed in } t] \end{Bmatrix}. \tag{40}$$

When a household retires, it retains its employment history, which is provided to its descendent. Thus, each new entrant initially projects its future employment based on its parent's history, then increasingly updates its history with its own experience.

5.1.3 Firms

To explore the relationship described in equation (36), we ensure cross-sectional variation in the labor share by assigning each firm an initial target number of employees derived from the share function: $f \Big/ \sum_{f=1}^{F} f$. At the start of each time period, each firm decides whether to increase or decrease its price for goods. Firms must perpetually search for optimal price using a simple algorithm. Firms make their pricing decision by assessing a running record of profits[7] for the previous N_f periods, and altering their strategies for *scaling* their prices. At the start of each period, each firm calculates *recent* profits $\sum_{t=1}^{N_f/2} \pi_{-t}$ and *bygone* profits $\sum_{t=(N_f/2)+1}^{N_f} \pi_{-t}$. The firm also knows its scaling strategy from the previous period, represented by a scaling factor $\delta_{-1} = \frac{p^*_{-1}}{p^*_{-2}} \in (0,2]$, where $\delta_t \in (0,1)$ denotes a price decrease in period *t*, and $\delta_t \in (1,2]$ denotes a price increase.

The firm chooses in each period whether to (1) *re-adopt* its strategy by applying the scaling factor used in the previous period: $p^*_0 \equiv \delta_0 \cdot p^*_{-1}$, where $\delta_0 \equiv \delta_{-1}$, or (2) *reverse* its strategy by reverting to the price associated with the highest profits in memory: $p^*_0 \equiv p^*_t \sim \max\{\pi_t\}_{t=-1}^{-N_f}$. In this latter case, the firm notes its reversal by resetting its pricing strategy: $\delta_0 = \frac{p^*_0}{p^*_{-1}}$.

In addition to an adaptive pricing strategy, the pricing algorithm also allows for two corrective adjustments. First, if a firm did not change its price in the previous period, then the firm tests its current strategy by randomly either increasing or decreasing its price by one-half percent. Second, if $p_{-1} > 0$ and $q^{sold}_{-1} = 0$, then the firm reduces price by one percent.

In simulations that allow for labor scaling, firms employ a similar but slightly different algorithm to search for optimal number of employees. We conducted two

---

[7] For purposes of generality, firms in this simulation search for the price that maximizes *real* profits, defined as $\chi \equiv \pi \big/ p$, which remains consistent with the conditions of equation **Error! Reference source not found.**.

different sets of simulations, without and without labor scaling. Table 4 lists the input parameters used in these simulations.

Table 4. Simulation Parameters

	Parameters	Symbol	Value
Global			
	Number of time periods	-	2000
	Number periods per year	λ	5
	Wage rate	w	50
	Market interest rate	r	5.0%
Households			
	Number of households	H	301
	Consumption-elasticity of utility	β	0.3
	Price-sensitivity exponent	γ	{-2,-3,-4,-5,-9}
	Discount rate	d	4.0%
	Minimum employment age	$Age_{\min}$	20
	Mandatory retirement age	Age_{retire}	60
	Expiration age	$Age_{\max}$	80
	Age distribution (uniform)	$\{Age_{h,t=0}\}_H$	~[20,80]
	Employment-eligible households	E	201
	Length of memory (# periods)	N_h	$3 \cdot \lambda$
Firms			
	Number of firms	F	5
	Productivity rate	ρ	2
	Initial labor share	$\sigma_{f,t=0}$	$f \Big/ \sum_{f=1}^{F} f$
	Initial reserves	$R_{f,t=0}$	$200K
	Length of memory (# periods)	N_f	$2 \cdot \lambda$

5.2 Results

We conducted two different sets of simulations, without and without labor scaling. Different simulations within each set of runs vary by the value of the households' price-sensitivity coefficient: γ.

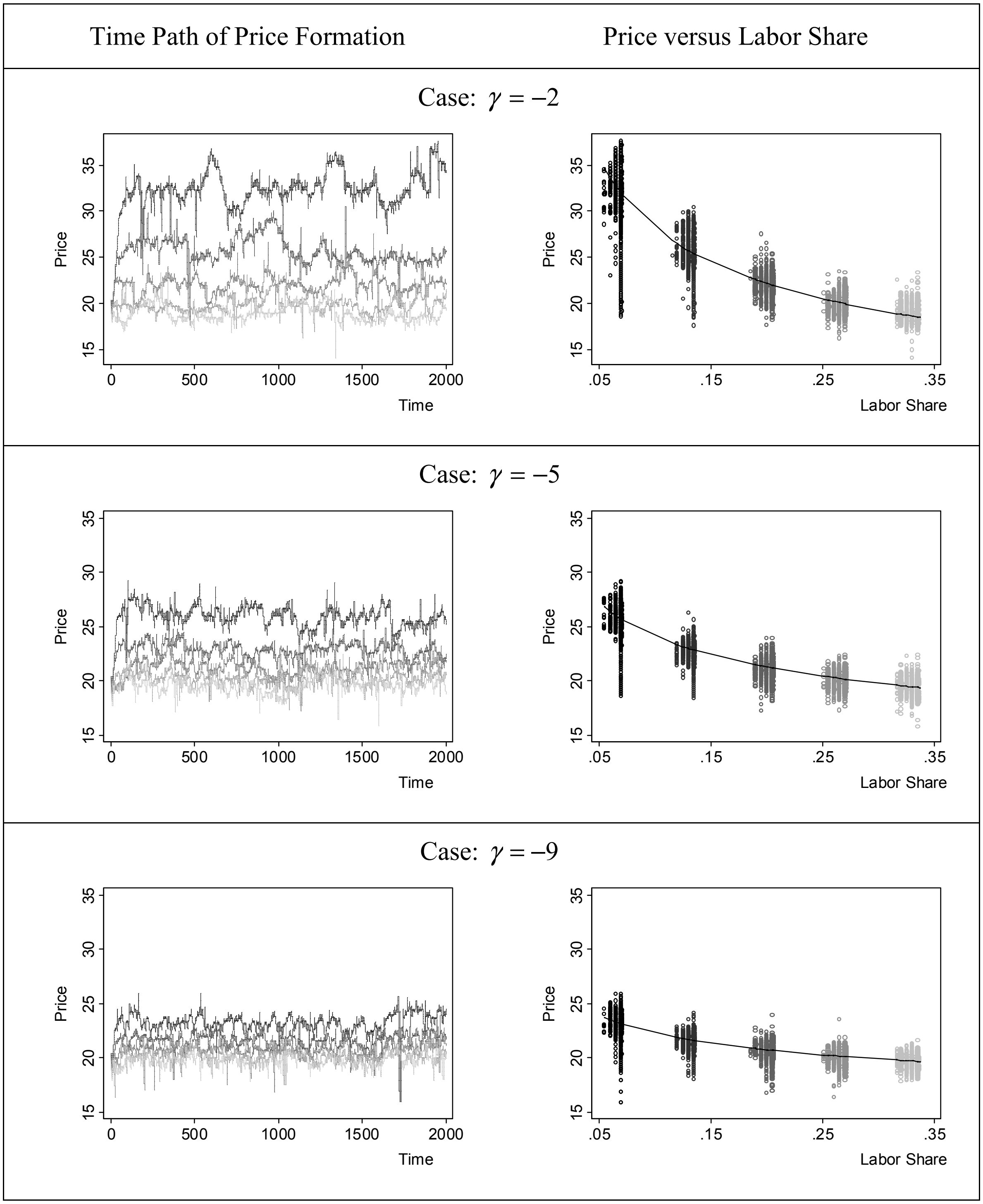
5.2.1 Set #1: Fixed Labor

The fixed-labor simulations are entirely deterministic, except for the quasi-rational firm selection rule. Firms employ rational adaptive pricing ($\nu = \infty$), and therefore deterministically (always) select the pricing strategy associated with the highest profits.

Firms are also restricted from laying off employees or from altering their initial target number of employees derived from the share function: $f \Big/ \sum_{f=1}^{F} f$. Thus, workers never experience layoffs and households' expectations are always formed by $y_t^e = w$ in equation (40). These baseline conditions provide both real and perceived job security for households, allowing them to optimize their time-dependent lifetime consumption.

By equation (36), we do not expect a single market equilibrium price, but rather a range of prices that vary across firms with respect to labor share as a function of γ. To demonstrate the robustness of convergence, we will run five baseline simulations assuming five different price-sensitivity exponents: γ = -2, -3, -4, -5, and -9. Figure 23 shows price formations for three of the five assumed values of γ. The left column shows the formation of each firm's price over time. The right column shows a projection of firms' prices onto a scatter plot with respect to the firms' labor shares. Each scatter plot also includes a curvilinear line-of-fit showing the log-linear relationship between prices and labor share. These plots demonstrate that (1) smaller firms converge to higher prices, (2) increasing consumer price sensitivity reduces both the average price and the cross-firm deviation in prices, and (3) prices form a log-linear relationship with labor share. Consistent with theory, these outcomes show that increasing consumer price sensitivity reduces firms' ability to assert market power, and thereby forces Nash equilibrium prices down closer to the competitive equilibrium.

Figure 23. Nash Equilibrium Price Formation Corresponding to Five Competing Firms with Different Labor Shares (i.e. numbers of employees).



5.2.2 Set #2: Adaptive Labor Scaling

The third set of simulations relaxes the baseline assumption that labor is fixed. Here, firms apply the adaptive labor scaling algorithm described in section 5.1.3. This algorithm allows for layoffs, which subsequently affect households' expectations and consumption profiles, and importantly, how they purchase in the goods market.

5.2.2.1 Effects of Labor Scaling on Pricing

Labor adjustments cause labor shares to change over time. Figure 24 compares price formation under labor adjustments with the baseline plots copied from Figure 23. These plots show that firms' labor shares and size rankings change substantially under labor scaling. However, the plots retain the baseline characteristics that (1) smaller firms achieve higher prices, (2) price deviation declines as price sensitivity increases, and (3) prices form a log-linear relationship with labor share.

5.2.2.2 Unemployment and Consumer Confidence

The adaptive labor search creates layoffs and an expectation of future layoffs, which causes currently or recently unemployed households to adjust consumption and savings accordingly. We can observe these adjustments in this simulation by comparing the households' financial profiles with those in the baseline simulation, in which households enjoyed both real and perceived job security.

Figure 25 shows the average household financial profile with respect to age assuming $\gamma = -2$ and $\nu = \infty$. In this exercise, all households have a fixed positive discount rate, resulting in greater planned consumption in earlier years. The upper plot shows the household's optimal planned consumption expenditure. Younger households achieve the optimal consumption by borrowing against future earnings. The lower plot shows the corresponding planned bank transactions required to achieve the consumption schedule shown in the upper plot. We see that each household will borrow loans through age 38, make loan payments and deposits from age 38 to 60, and make withdraws after retirement at age 60. These plots show that adaptive labor search causes wary households restrict consumption in the earliest stages of the life cycle (upper plot) to reduce their debt stream (lower plot).

In this simulation, households form "rational" expectations of a sort by incorporating their employment history into their expectation for future income. For further comparison, we executed an additional simulation in which households were fully optimistic concerning future employment regardless of their consumption history. Figure 25 shows that optimistic households borrow more resulting in lower consumption during both their career and their retirement. This demonstrates that firms' search for optimal production scale creates non-cyclical unemployment, which affects the households' financial profiles. The nature and magnitude of those effects, however, depend on households' memory of and attitude toward unemployment.

Figure 24. Price Formation with Fixed versus Adjusted Labor Corresponding to Five Firms with Different Initial Labor Shares

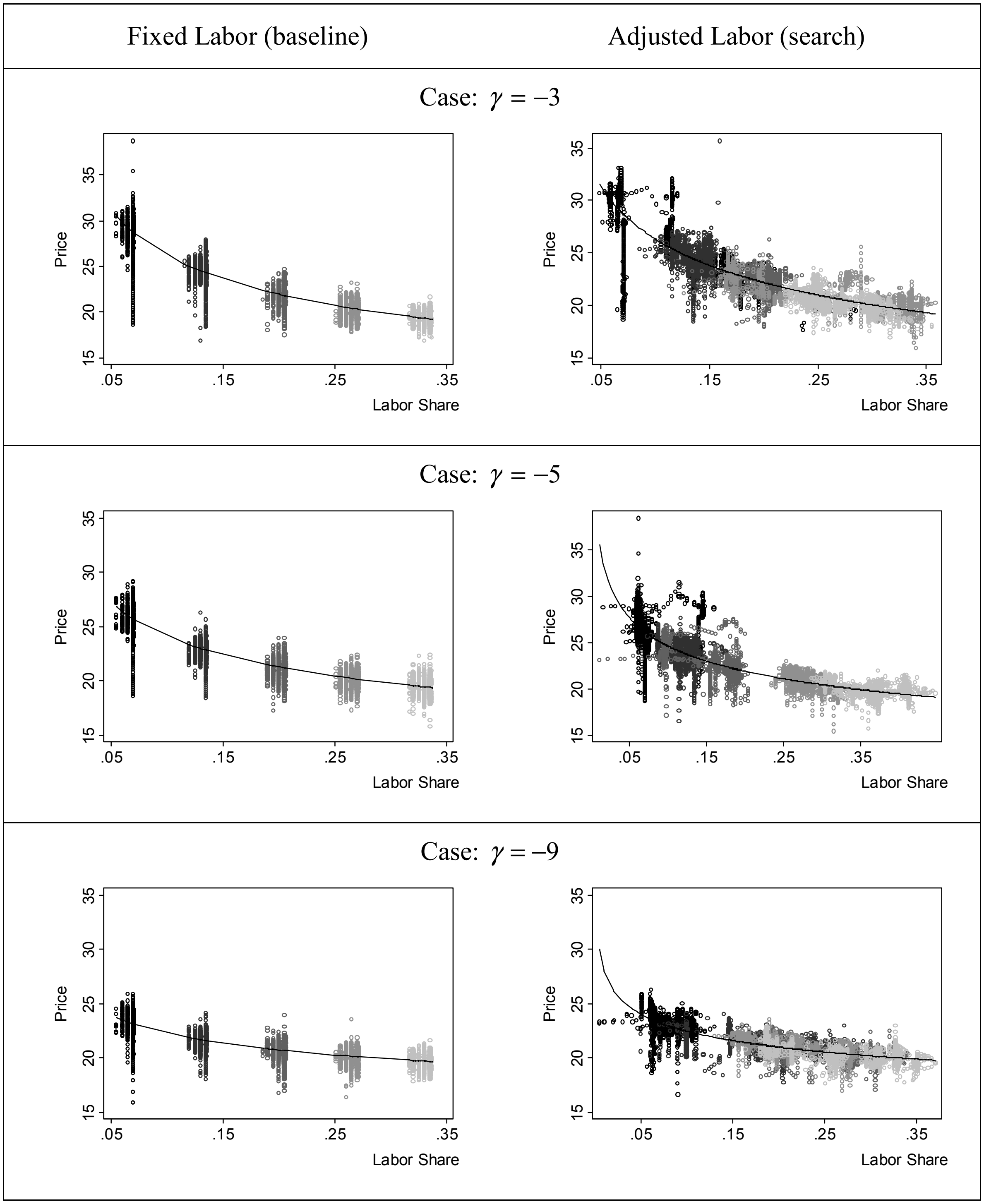
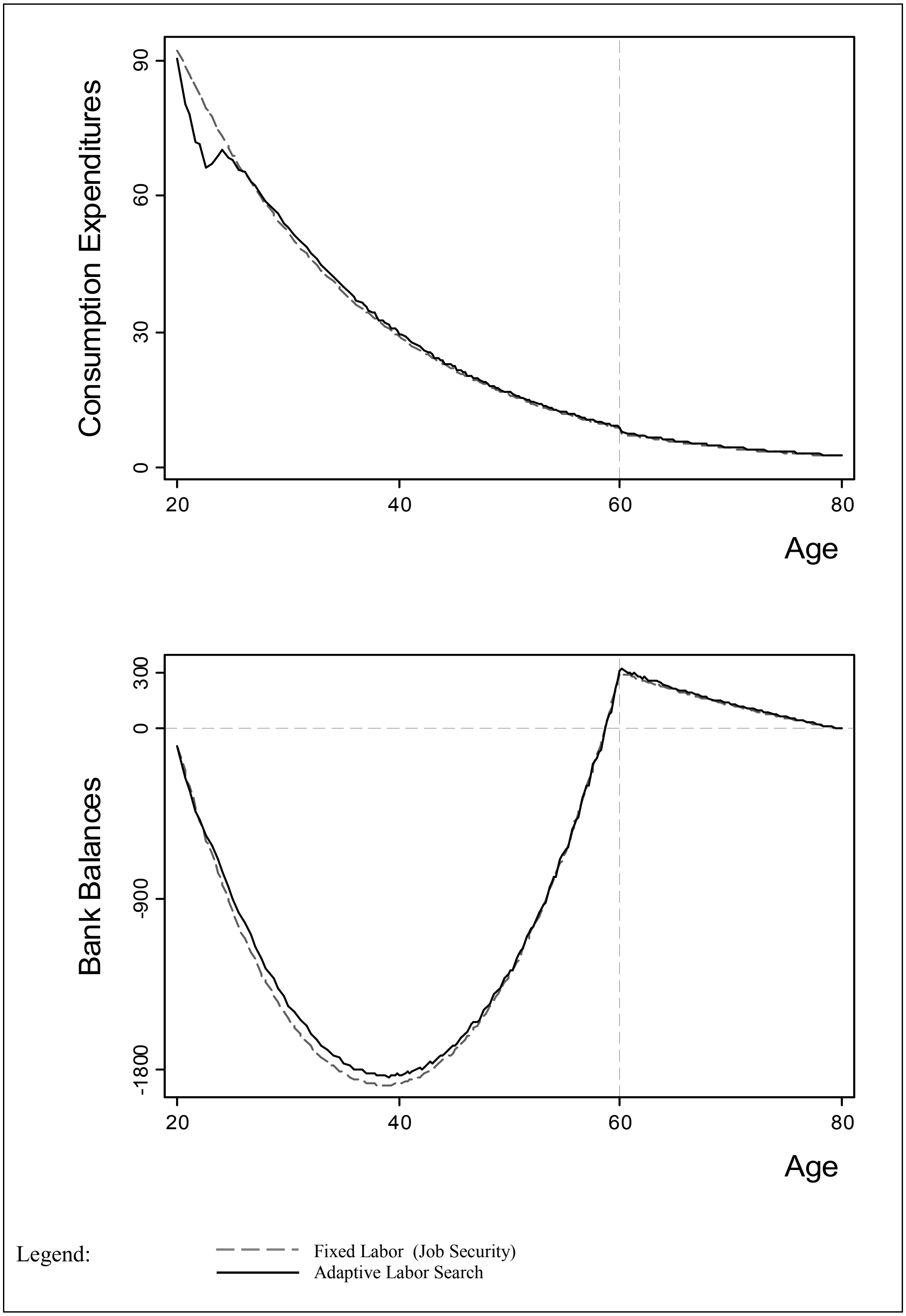


Figure 25. Unemployment and Consumer Confidence



5.3 Conclusions

This exercise demonstrates the robustness of theory in a complex system of interrelated agents and markets. Specifically, we find that firms with limited memory, no public information, and very simplistic decision processes can "discover" their Nash equilibrium prices despite various sources of noise and uncertainty. The firms converge despite relative ignorance and a reliance on extremely rudimentary search algorithms. Thus, the simulation supports the robustness and validity of the analytical model.

This exercise models a link between firm decisions and national savings and consumption rates, and therefore provides a foundation to analyze how events that evoke corporate and industrial reaction can have cascading impacts on confidence and the economy.

6 An Agent-Based Modeling Framework

The inconclusive empirical findings, discussed in Section 2, regarding the role of confidence suggest the need for alternatives to conventional analytical and empirical methods. We have introduced models implemented as agent-based simulations for that purpose. Agent-based economic simulation is a computational approach for integrating models of social choice into complex systems of artificial decision makers that allow researchers to conduct controlled economic experiments. The methodology involves the use of computer programs to distribute information, decisions, and communications across many well-defined economic participants who follow certain rules while trying to optimize their user-defined objectives (e.g. utility functions). The experimenter's objective is to replicate the relevant economic activity of individuals, and thereby study complex collective behaviors. We here present a discussion of the agent simulation framework we used for the analytic analysis.

6.1 Characteristics of a General Modeling Framework

Object-oriented programming (OOP) is a natural, and hitherto dominant, framework for building agent-based simulations. Examples of OOP languages include C^{++}, Objective C, and Java/$C^{\#}$. Luna and Stefansson (2000, see Introduction) attribute the advantages of OOP to four properties of OOP: abstraction, encapsulation, inheritance, and polymorphism. Luna and Stefansson state, "these properties allow the programmer to conceive an agent as a self-contained (encapsulated) object which is the `tangible´ instance of some initial template (abstraction), and which has inherited some general features that define its essence without `hindering´ its potential development." We retain the object-oriented paradigm as a foundation for the following proposed features.

6.1.1 Modular

Stemming of from principles of object-oriented source code, modularization extends to the broader concept of a flexible modeling environment. Specifically, agents and their internal functions and behaviors must be modularized for drag-n-drop visual programming. Modules are defined by their underlying source-code class, but also by there placement and relation in

6.1.2 Hierarchical

A general framework must allow for hierarchical modular networks, in which a module can contain a collection of sub-modules, and a network can contain a collection of sub-networks.

6.1.3 Visual Interface

A general framework requires a visual design interface conforming to the visual drag-n-drop paradigm employed by many discrete-event and systems-dynamics software packages.

6.2 Features of a General Modeling Framework

6.2.1 Network Flow

Users should be able to explicitly define data flows, a common need within for discrete-event and systems-dynamics software packages. Users specify output ports to which a module writes data, incoming ports from which the module reads data, and links which define the flow of data from one module to another.

6.2.2 Messaging

The framework has an implicit messaging system. Modules will be endowed with an inbox and outbox for reading and writing messages. The messaging system will route messages between agents. Messages can be routed according to several protocols. Private messages are sent directly to one or more specified agents. Disseminations are sent directly to a specified module, which them forward to message to all of its sub-modules. Bulletins are sent to a special module called a bulletin board, which posts the message for access by all modules that have subscribed to the bulletin board.

6.2.3 Cloning

Agent models often require many instantiations of each class of agent. Visual interfaces must allow users to hide many agents within a single agent-module simply for managing visual complexity. Cloning allows a module to be replicated a specified number of times at run-time. It also allows the parameters of the cloned module to be varied across clones according to user-specified distributions. Cloning can be very general, allowing for the cloning of any model structure or class.

6.2.4 Spontaneous Restructuring

This refers to run-time modification of instantiated objects based on the rules or functions defined within agents or environment. Unlike the *scheduled event* capabilities common in discrete-events packages, spontaneous restructuring is not necessary planned nor predictable.

For our purposes, spontaneous restructuring can take several forms. One example is the run-time instantiation or destruction of agents. Another example is the run-time instantiation or destruction of sub-agents or objects contained within agents. A third example is the run-time creation or destruction of data flows.

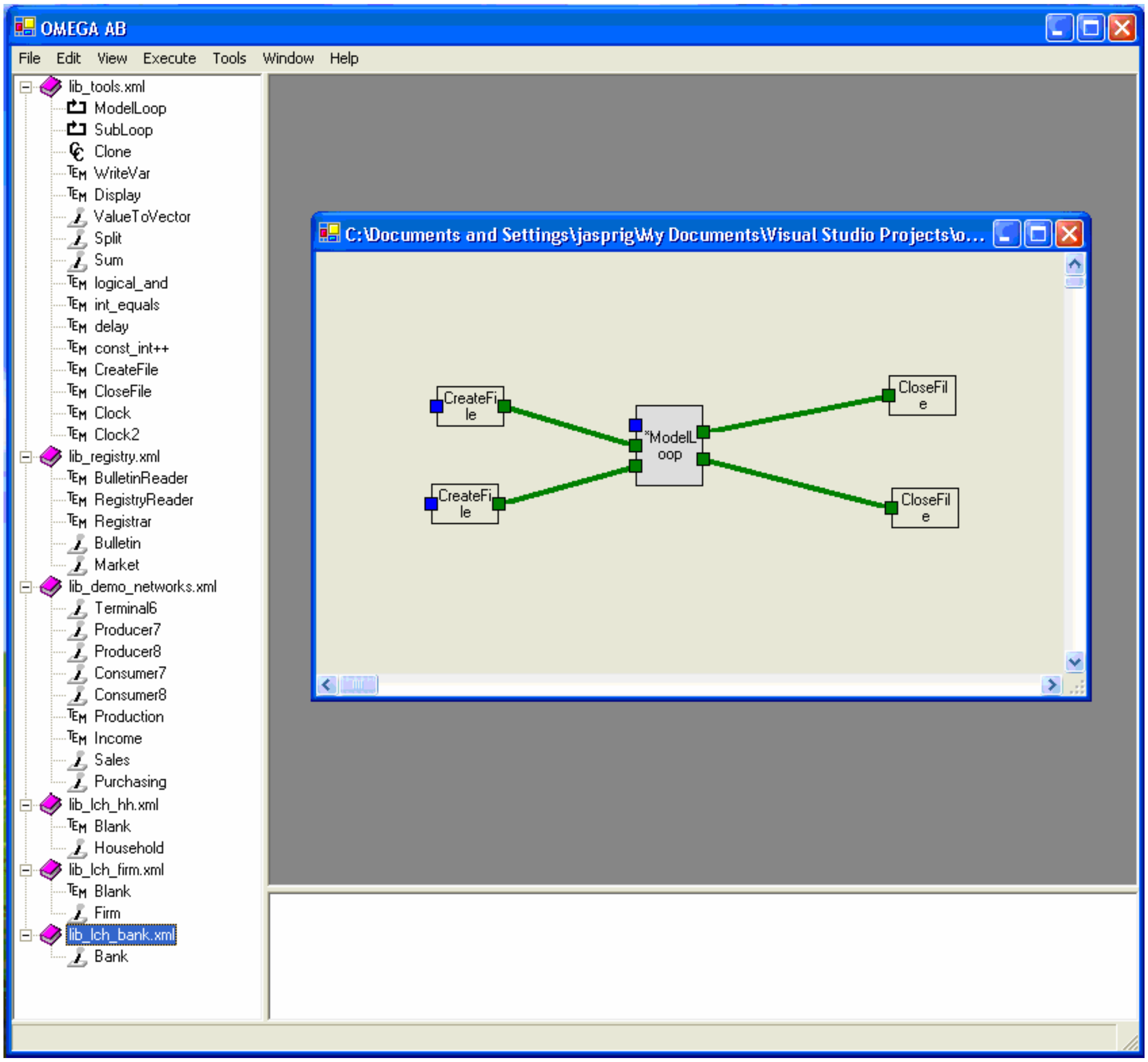
6.3 Economic Simulation in the General Framework

We now introduce a new framework comprised of a graphical front-end, an associated interpreter, and a collection of XML and C++ module libraries. The user graphically places and connects modules, found in the libraries, to create “programs”. The programs are saved to an XML file and sent to the interpreter for execution.

6.3.1 Visual Interface

We implemented the economic simulation from section 5 in the new framework. Figure 26 shows the visual interface for the new capability. Shown on the left is a tree view of predefined module libraries. The landscape (on the right) shows the top-level simulation wrapper containing five modules. The two leftmost modules are used to specify paths and filenames for two files to be opened upon execution. The file streams for these files are passed via explicit port links to the "Model Loop" module, which contains the economic model. The file stream are then pass out of the Model Loop to the rightmost modules, which will terminate the file stream and close the files upon completion of the simulation.

Figure 26. Visual Interface showing the top layer of Economic Simulation



6.3.2 Modularization

Each of the modules listed in the libraries on the left have metadata to specify how the module is to be interpreted by the simulation engine, including class, port and data flow information, and module-specific parameters.

Any of the modules listed in the libraries on the left could be selected and either dragged-n-dropped or copied-n-pasted into the landscape for inclusion in the simulation. Once a module resides in the landscape, some of its metadata are visually displayed to the user, such as the visual display of input and output data ports. Other metadata can be accessed by right-clicking on the module to obtain module properties.

6.3.3 Hierarchy

A simple demonstration of hierarchy occurs when the user clicks on the Model Loop module, which opens the window shown in Figure 27, which shows the agent network contained within that module.

The model shows four firms, a bank, and a "households" module. The *Households* module is a clone wrapper, which contains a single household module. Upon execution, the *Households* module creates N clones of the household, effective creating a model of N households. Other modules exist to facility market transactions. For example, the *Market* module serves as an exchange allowing firms to post product offers, which are made available to households that subscribe to the market's bulletins.

Some modules provide analytical capability to the user, rather than serving as model components. For example, the *WriteVar* module accept user define data inputs, and write those inputs to a user-defined file stream for post-simulation assessment and analysis.

The tool allows for hierarchical data flow. Each module contains two modules for accessing input and output data ports. For example, the leftmost module below contains three data ports corresponding to the three input ports shown on the *Model Loop* module in Figure 26.

Figure 27. Model Loop containing Economic Simulation

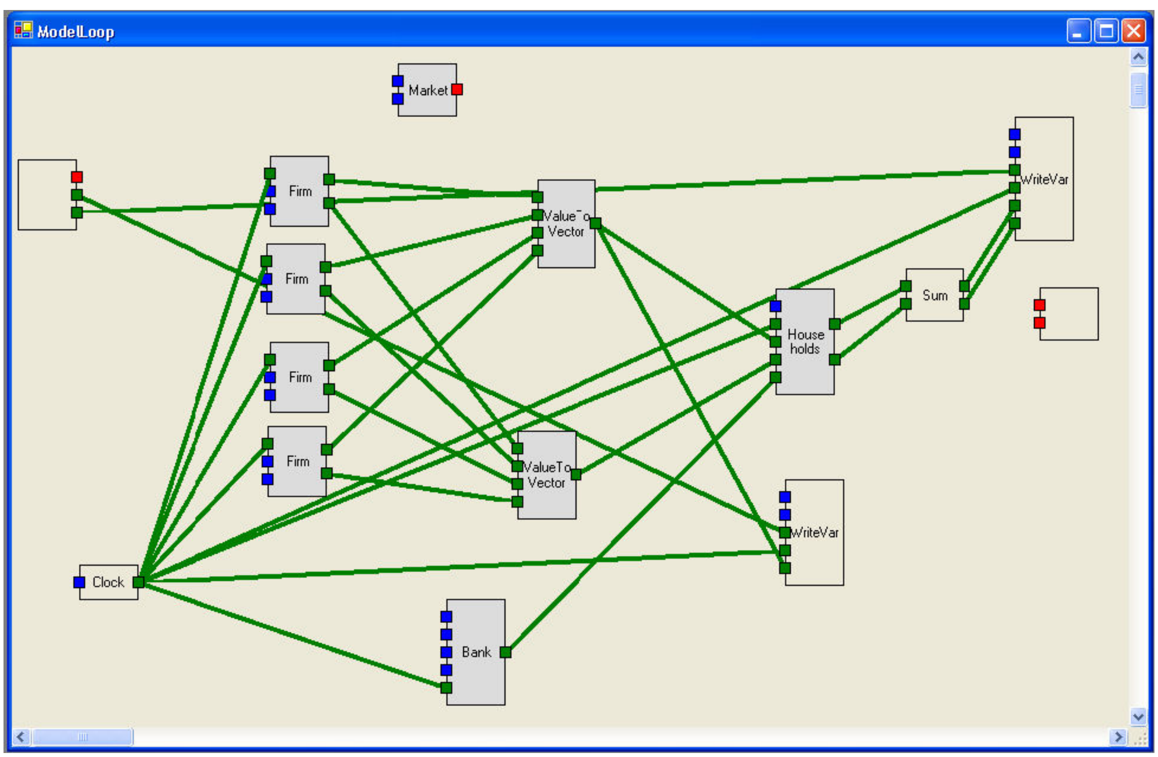
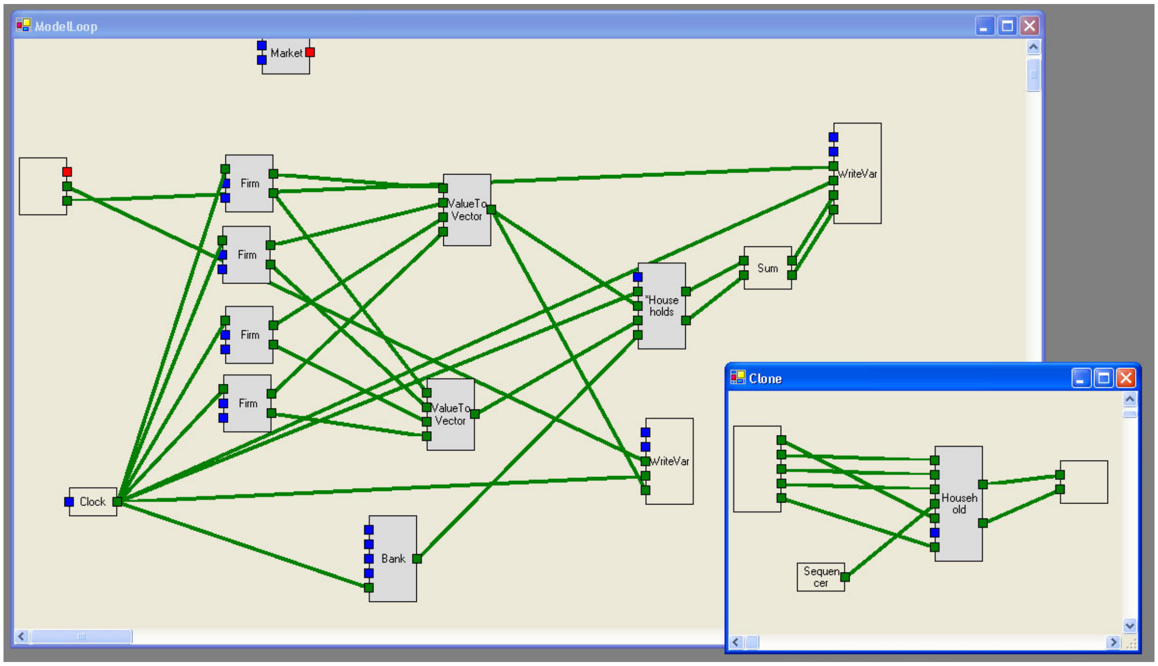


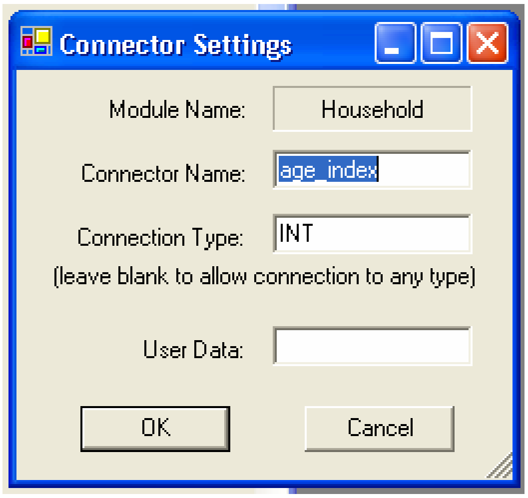
Figure 28 shows the use of a clone wrapper for instantiating many agents of a class. Clicking on the “Households” module (which is a clone wrapper) in the primary window opens the window shown in the lower right of Figure 28. This window displays the two modules contained in the clone wrapper: a “Household” class and a “sequencer” class. At configuration time, the simulation engine instantiates N households.

Figure 28. A Clone Wrapper for Household Agents



Our overlapping generation model calls for a uniform age distribution for households. To vary age in the visual framework, we first specify an “age index” as an input parameter to the household. This allows the user to specify the age index for each household agent at design time. Upon instantiation, each household class will execute an internal calculation to translate the age index into an initial value for its internal age variable. In Figure 28, the age index parameter is the 4^{th} input port on the left of the household module. Right-clicking at this port will display a “settings” window shown in Figure 29. Note that this port is connected to the “sequencer” module. The sequencer causes the initial value to be incremented across clones at instantiation time causing a uniform distribution of age.

Figure 29. Settings for the “age index” input parameter.



These descriptions show the kind of software components used to visually create a modular hierarchical the economic simulation presented in this report.

6.3.4 Summary

The use of logic and framework described here enhances the ability to simulate interesting agents and analyze disruptive economic events. The framework is not only flexible to support the changing needs of the individual developer or modeler, but it also eases collaboration among many analysts and provides natural and accessible documentation of all levels and stages from sub-agent modules to complex multi-agent systems of systems.

7 Remarks

We have implemented a complex agent-based economic model with confidence-related components in a visual, modular, hierarchical framework. This effort has provided a foundation for both economic analysis of the potential impacts of terrorism on confidence and the economy, but it also provides a foundation for a general framework for such models.

This work indicates the efficacy of using agent-based simulation to analyze the impact of terrorist events on confidence and the economy. The results shown here provide a foundation for future studies on behavioral and economic dynamics, as they relate to the recovery of the economy after terrorist events.

8 References

Abel, Andrew B. 1985. "Precautionary Savings and Accidental Bequest." *American Economic Review* 75 (September): 777-791.

_______. 2001. "Will Bequests Attenuate the Predicted Meltdown in Stock Prices When Baby Boomers Retire?" NBER Working Paper 8131 (February).

Acemoglu, Daron and Andrew Scott. 1994. "Consumer Confidence and Rational Expectations: Are Agents' Beliefs Consistent with the Theory?" *The Economic Journal* 104 (January): 1-19.

Akerlof, George. 1970. "The Market for Lemons: Quality Uncertainty and the Market Mechanism." *Quarterly Journal of Economics* 84 (August): 488-500.

Alexandridis, Konstantinos T., and Bryan C. Pijanowski. 2002. "Multi Agent-Based Environmental Landscape (MABEL) - An Artificial Intelligence Simulation Model: Some Early Assessments." Presented at the Second World Congress of Environmental and Resource Economics. (June).

Anderson, Philip W., Kenneth J. Arrow, and David Pines. 1988. *The Economy as an Evolving Complex System*. Redwood City: Addison Wesley.

Ando, Albert, and Modigliani, Franco. 1963. "The 'Life Cycle' Hypothesis of Saving: Aggregate Implications and Tests." *American Economic Review* 53 (March): 55–84.

Anselin, Luc. 2003. "Spatial Externalities, Spatial multipliers, and Spatial Econometrics." *International Regional Science Review* 26 (April): 153-166.

Arrow, Kenneth. 1971. *Essays in the Theory of Risk Bearing*. Chicago: Markham Publishing Company.

Arthur, Brian W. 1995. "Complexity in Economics and Financial Markets." *Complexity* 1 (April).

___________, Steven N. Durlauf, and David A. Lane. 1997. *The Economy as an Evolving Complex System II*. Redwood City: Addison Wesley.

Arthur, Brian W. 1999. "Complexity and the Economy." *Science* 284 (April): 107-109.

Axelrod, Robert. 1997. *The Complexity of Cooperation: Agent-Based Models of Competition and Collaboration*. Princeton: Princeton University Press.

Balasko, Yves, and Karl Shell. 1980 "The Overlapping-Generations Model, I: The Case of Pure Exchange without Money." *Journal of Economic Theory* 23 (December): 281-306.

Balasko, Yves, David Cass, and Karl Shell. 1980 “Existence of Competitive Equilibrium in a General Overlapping-Generations Model.” *Journal of Economic Theory* 23 (December): 307-322.

Balasko, Yves, and Karl Shell. 1981 “The Overlapping-Generations Model, II: The Case of Pure Exchange with Money.” *Journal of Economic Theory* 24 (February): 112-142.

______________. 1981 “The Overlapping-Generations Model, III: The Case of Log-Linear Utility Functions.” *Journal of Economic Theory* 24 (February): 143-152.

Baldini, Massimo. 2001. “Inequality and Redistribution over the Life-Cycle in Italy: an Analysis with a Dynamic Cohort Microsimulation Model.” *Brazilian Electronic Journal of Economics* 4 (December).

Bankes, Steven, Robert Lempert, and Steven Popper. 2002. “Making Computational Social Science Effective: Epistemology, Methodology, and Technology.” *Social Science Computer Review* 20 (Winter): 377-88.

Barro, Robert J. 1974. “Are Government Bonds Net Wealth?” *Journal of Political Economy* 82 (November): 1095-1118.

___________. 1976. “Rational Expectations and the Role of Monetary Policy.” *Journal of Monetary Economics* 2: 1-32.

___________. 1978. “Unanticipated Money, Output and the Price Level in the United States.” *Journal of Political Economy* 86 (August): 548-580.

___________. 1979. “On the Determination of Public Debt.” *Journal of Political Economy* 87 (October): 940-971.

___________. 1984. “Rational Expectations in Macroeconomics in 1984.” *The American Economic Review* 74 (2): 179-182.

___________. 1991. “This Week's Citation Classic: 'Rational expectations and the role of monetary policy'.” Current Contents, *Arts & Humanities and Social Sciences* 26: 22.

Barton, Dianne C., Eric D. Eidson, David A. Schoenwald, Kevin L. Stamber, and Rhonda K. Reinert. 2000. *Aspen-EE: An Agent-Based Model of Infrastructure Interdependency*. SAND2000-2925. Albuquerque, NM: Sandia National Laboratories.

Barton, Dianne C., Eric D. Eidson, David A. Schoenwald, Roger R. Cox, and Rhonda K. Reinert. 2004. *Simulating the Effects of Disruptions in the Telecommunications Infrastructure*. SAND2004-0101. Albuquerque, NM: Sandia National Laboratories.

Basu, Nipa, Richard J. Pryor, and Tom Quint. 1998. “ASPEN: A Microsimulation Model of the Economy.” *Computational Economics* 12 (December): 223–241.

Basu, Nipa, and Richard J. Pryor. 1997. *Growing a Market Economy* SAND1997-2093. Albuquerque, NM: Sandia National Laboratories.

Batchelor, Roy and Pami Dua. 1998. “Improving Macro-economic Forecasts: The Role of Consumer Confidence,” *International Journal of Forecasting* 14 (March): 71-81.

Bearce, David H. and Eric Fisher. 2002. “Economic Geography, Trade, War.” *Journal of Conflict Resolution* 46 (June): 365-393.

Becker, Gary S. and Gilbert R. Ghez. 1975. *The Allocation of Time and Goods Over the Life Cycle*. New York: NBER and Columbia Press.

Bergman, Aviv, and Moshe Tennenholtz. 2002. “On the Natural Selection of Market Choice.” *Autonomous Agents and Multi-Agent Systems* 5 (December): 385-395.

Bernstein, Peter L. 1998. *Against the Gods: The Remarkable Story of Risk*. New York: John Wiley & Sons, Inc.

Bertels, Koen, and Magnus Boman. 2001. “Agent-Based Social Simulation in Markets.” *Electronic Commerce Research* 1 (February): 149-158.

Bird, Grahm. 2002. "What difference does it make? The global economic effects of international terrorism." *New Economy* 9 (June): 106-112.

Bordini, Rafael H., Ana L. C. Bazzan, Rosa M. Vicari and John A. Campbell. 2000. “Moral Sentiments in the Iterated Prisoner's Dilemma and in Multi-Agent Systems.” *Brazilian Electronic Journal of Economics* 3 (June).

Bowden, Roger. 1974. “Risk Premiums and the Life Cycle Hypothesis.” *American Economic Review* 64 (1): 211-216.

Bram, Jason and Sydney Ludvigson. 1998. “Does Consumer Confidence Forecast Household Expenditure? A Sentiment Index Horse Race.” Federal Reserve Bank of New York: *Economic Policy Review* 4 (June): 59-78.

Buchta, Christian, David Meyer; Alexander Pfister; Andreas Mild; Alfred Taudes. 2003. “Technological Efficiency and Organizational Inertia: A Model of the Emergence of Disruption.” *Computational & Mathematical Organizational Theory* 9 (July): 127-146.

Bullard, James and James Feigenbaum. 2004 (revised). “A Leisurely Reading of the Life-Cycle Consumption Data.” Federal Reserve Bank of St. Louis Working Paper 2003-017C. St. Louis, MO: Federal Reserve Bank of St. Louis.

Campbell, John Y. and N. Gregory Mankiw. 1990. “Permanent Income, Current Income, and Consumption,” *Journal of Business and Economic Statistics* 8 (July): 265-279.

Carpenter, Jeffrey P. 2002. "Evolutionary Models of Bargaining: Comparing Agent-based Computational and Analytical Approaches to Understanding Convention Evolution." *Computational Economics* 19 (February): 25-49.

Carroll, Christopher D. 1997. "Buffer-Stock Saving and the Life-Cycle/Permanent Income Hypothesis." *Quarterly Journal of Economics* 112 (1): 1-55.

Cedarman, Lars-Erik. 2002. "Endogenizing Geopolitical Boundaries with Agent-Based Modeling." *Proceedings of the National Academy of Sciences* 99 (May) 7296-7303.

Chaturvedi, Alok, and Shailendra Mehta. 1999. "Simulations in Economics and Management: Using the SEAS Simulation Environment." *Communications of the Association for Computing Machinery* 42 (March): 60-61.

Chaturvedi, Alok, Shailendra Mehta, Daniel Dolk, and Rick Ayer. 2004. "Agent-Based Simulation for Computational Experimentation: Developing an Artificial Labor Market." *European Journal of Operations Research*.

Chaturvedi, Alok, Shailendra Mehta, and Paul Drenevich. 2004. "Computational and Live Experimentation in Bio-terrorism Response." in *Dynamic Data Driven Applications Systems*. F. Darema (ed). Amsterdam: Kluwer Publications. 1-16.

Chen, Shu-Heng. 2002. Evolutionary Computation in Economics and Finance. New York: Springer.

Chen, Shu-Heng, and Chia-Hsuan Yeh. 2002. "On the Emergent Properties of Artificial Stock Markets: The Efficient Market Hypothesis and the Rational Expectations Hypothesis." *Journal of Economic Behavior and Organization* 49 (October): 217-239.

Danielson, Peter. 2002. "Competition Among Cooperators: Altruism and Reciprocity." *Proceedings of the National Academy of Sciences* 99 (May) 7237-7242.

DeCanio, Stephen J., and William E. Watkins. 1998. "Information Processing and Organizational Structure." *Journal of Economic Behavior and Organization* 36 (August): 275-294.

Desroches, Brigitte and Marc-Andre Gosselin. 2002. The Usefulness of Consumer Confidence Indexes in the United States. Bank of Canada Working Paper 2002-22. Ottawa, Ontario: Bank of Canada.

Deveaux, Laurent, Corina Paraschiv, Mathieu Latourrette. 2001. "Bargaining on an Internet Agent-based Market: Behavioral vs. Optimizing Agents." *Electronic Commerce Research* 1 (October): 371-401.

Domowitz, Ian, and Jianxin Wang. 1994. "Auctions as Algorithms: Computerized Trade Execution and Price Discovery." *Journal of Economic Dynamics and Control* 18 (1): 29-60.

Drenevich, Paul, Shailendra Mehta, Alok Chaturvedi and Thomas Brush. 2004. “Large Scale Agent-based Simulations – An Illustrative Application: Coordinated Government Agency Response to Bio Terrorism.” Working paper.

Durlauf, Steven N., H. Peyton Young, eds. 2001. *Social Dynamics (Economic Learning and Social Evolution)*. Cambridge, MA: MIT Press.

Edmonds, Bruce. 2001. “Towards a Descriptive Model of Agent Strategy Search.” *Computational Economics* 18 (August): 111-133.

Ehlen, Mark A., and Robert J. Glass Jr. *Growth and Defection in Hierarchical, Synergistic Networks*. SAND2003-3194C. Albuquerque, NM: Sandia National Laboratories.

Elliott, Euel, and L. Douglas Kiel. 2002. Exploring Cooperation and Competition using Agent-Based Modeling.” *Proceedings of the National Academy of Sciences* 99 (May) 7193-7194.

Eppright, David R., Nestor M. Arguca, and William L. Huth. 1998. “Aggregate Consumer Expectation Indexes as Indicators of Future Consumer Expenditures.” *Journal of Economic Psychology* 19 (April): 215-235.

Epstein, Joshua. 2002. “Modeling Civil Violence: An Agent-Based Computational Approach.” *Proceedings of the National Academy of Sciences* 99 (May) 7243-7250.

___________, and Robert Axtell. 1996. *Growing Artificial Societies*. Cambridge, MA: MIT Press.

Farmer, J. Doyne. 2001. “Toward Agent-Based Models for Investment.” In *Developments in Quantitative Investment Models*. Association for Investment Management and Research.

______________, Paolo Patelli, and Ilija Zovko. 2003. “The Predictive Power of Zero Intelligence Models in Financial Markets.” Los Alamos National Laboratory Condensed Matter Abstract.

Fisher, Irvin. 1930. *The Theory of Interest: As Determined by Impatience to Spend Income and Opportunity to Invest It*. New York: Macmillan Company.

Flavin, Marjorie A. 1981. “The Adjustment of Consumption to Changing Expectations about Future Income.” *Journal of Political Economy* 89 (October): 974-1008.

Friedman, Milton. 1953. *Essays in Positive Economics*. Chicago IL: University of Chicago Press.

___________. 1957. *A Theory of the Consumption Function*. Princeton NJ: Princeton University Press.

___________. 1977. "Nobel Lecture: Inflation and Unemployment." *Journal of Political Economy* 85 (June): 451-471.

Garner, C. Alan. 1991. "Forecasting Consumer Spending: Should Economists Pay Attention to Consumer Confidence Surveys?" Federal Reserve Board of Kansas City: *Economic Review* (May/June): 57-71.

___________. 2002. "Consumer Confidence After September 11." Federal Reserve Bank of Kansas City: *Economic Review* (Second Quarter): 5-25.

Gilbert, N., and P. Terna. 2000. "How to Build and Use Agent-Based Models in Social Science." *Mind & Society* 1: 57-72.

Grossman, Sanford J., and Oliver D. Hart. 1983. "An Analysis of the Principal-Agent Problem." *Econometrica* 51 (January): 7-45.

Gupta, Mukul, Shailendra Mehta and Alok Chaturvedi. 2003. "Synthetic Environment for Analysis and Simulation: A Case Study of Market and Supply Chain Co-Design in the PC Industry." Working paper.

Hand, Michael S., Paul J. Paez, and James A. Sprigg Jr. 2005. *On the Need and Use of Models to Explore the Role of Economic Confidence: A Survey*. SAND2005-2445. Albuquerque, NM: Sandia National Laboratories.

Harsanyi, John. 1967. "Games with Incomplete Information Played by 'Bayesian' Players, I: The Basic Model." *Management Science* 14 (3): 159-182.

___________. 1968. "Games with Incomplete Information Played by 'Bayesian' Players, II: Bayesian Equilibrium Points." *Management Science* 14 (5): 320-334.

___________. 1968. "Games with Incomplete Information Played by 'Bayesian' Players, III: Basic Probability Distribution of the Game." *Management Science* 14 (7): 486-502.

Hayek, Friedrich. 1948. *Individualism and Economic Order*. Chicago: University of Chicago Press.

Heckman, James. 1974. "Life-Cycle Consumption and Labor Supply: An Explanation of the Relationship Between Income and Consumption Over the Life Cycle." *American Economic Review* 64 (1): 188-194.

Holland, John H. and John H. Miller. 1991. "Artificial Adaptive Agents in Economic Theory" *American Economic Review* 81 Papers and Proceedings of the Hundred and Third Annual Meeting of the American Economic Association (May): 365-370.

Holland, John H. 1992. *Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence*. Bradford Books.

___________. 1996. *Hidden Order: How Adaptation Builds Complexity*. Addison Wesley Publishing Company.

___________. 1999. *Emergence: From Chaos to Order*. Perseus Books Group.

Holmstrom, B. 1979. "Moral Hazard and Observability," *Bell Journal of Economics* 10 (Spring): 74-91.

Howrey, E. Philip. 2001. "The Predictive Power of the Index of Consumer Sentiment." *Brookings Papers on Economic Activity* 1: 175-216.

Hubbard, R. Glenn, Jonathan Skinner, and Stephen Zeldes. 1994. "The Importance of Precautionary Motives in Explaining Individual and Aggregate Savings." *Carnegie-Rochester Conference Series on Public Policy* 40 (June): 59-125.

Izumi, K. and K. Ueda. 2001. "Phase Transition in a Foreign Exchange Market-Analysis Based on an Artificial Market Approach." *IEEE Transactions on Evolutionary Computation* 5 (October): 456-470.

Janssen, Marco A., and Wander Jager. 2003. "Simulating Market Dynamics: Interactions between Consumer Psychology and Social Networks." *Artificial Life* 9 (Fall): 343-56.

Judd K. L., and L. Tesfatsion, eds. 2006. *Handbook of Computational Economics, Volume 2: Agent-Based Computational Economics*. Cambridge, MA: North-Holland.

Katona, George. 1975. *Psychological Economics*. NY: Elsevier Scientific Publishing Company.

___________. 1972. "Theory of Expectations." in Strumpel, Morgan, and Zahn ed. *Human Behavior in Economic Affairs: Essays in Honor of George Katona*. San Francisco: Jossey-Bass Inc.

Katz, Daniel. 1972. "Psychology and Economic Behavior," in Strumpel, Morgan, and Zahn ed. *Human Behavior in Economic Affairs: Essays in Honor of George Katona*. San Francisco: Jossey-Bass.

Kutschinski, Erich, Thomas Uthmann and Daniel Polani. 2003. "Learning Competitive Pricing Strategies by Multi-Agent Reinforcement Learning." *Journal of Economic Dynamics and Control* 27 (September): 2207-2218.

LeBaron, B. 2001. "Empirical Regularities from Interacting Long- and Short-Memory Investors in an Agent-Based Stock Market." *IEEE Transactions on Evolutionary Computation* 5 (October): 442-455.

Leijonhufvud, Axel. 1999. "Adaptive Behavior, Market Processes and the Computable Approach." *Brazilian Electronic Journal of Economics* 2 (June).

Levy, Horacio, Magda Mercader-Prats, and Mercedes Planas. 2001. "An Introduction to ESPASIM: A Microsimulation Model to Assess Tax-Benefit Reforms in Spain." *Brazilian Electronic Journal of Economics* 4 (September).

Lempert, Robert. 2002. "Agent-Based Modeling as Organizational and Public Policy Simulators." *Proceedings of the National Academy of Sciences* 99 (May): 7195-7196.

Lettau, Martin. 1997. "Explaining the Facts with Adaptive Agents: The Case of Mutual Fund Flows." *Journal of Economic Dynamics and Control* 21: 1117-1147.

Lucas, Robert. 1972. "Expectations and the Neutrality of Money." *Journal of Political Economy* 4 (2): 103-124.

Lucas, Robert, and Edward Prescott. 1971. "Investment Under Uncertainty." *Econometrica* 39 (September): 659-681.

Luna, Francesco, and Alessandro Perrone, eds. 2002. *Agent-Based Methods in Economics and Finance: Simulations in Swarm*. Boston: Kluwer Academic Publishers.

Luna, Francesco, and Benedikt Stefansson, eds. 2000. *Economic Simulations in Swarm: Agent-Based Modelling and Object Oriented Programming*. Boston: Kluwer Academic Publishers.

Macy, M., and A. Flache. 2002. "Learning Dynamics in Social Dilemmas." *Proceedings of the National Academy of Sciences* 99 (May) 7229-7236.

Marks, Robert E. 2001. "Playing Games with Genetic Algorithms." in Chen ed. *Evolutionary Computation in Economics and Finance*. New York: Springer.

Maza, Michael de la, Ayla Ogus, and Deniz Yuret. 1998. "How Do Firms Transition Between Monopoly and Competitive Behavior? An Agent-Based Economic Model." *Proceedings of the Sixth International Conference on Artificial Life*: 349-357.

McCain, Roger. 2000. Agent-Based Computer Simulation of Dichotomous Economic Growth. Boston: Kluwer Academic Publishers.

McCandless, George T., Jr., and Neil Wallace. 1991. *Introduction to Dynamic Macroeconomic Theory: An Overlapping generations Approach*. Cambridge, MA: Harvard University Press.

McFadden, Daniel. 1974. "Conditional Logit Analysis of Qualitative Choice Behavior," in P. Zarembka (ed.) *Frontiers of Econometrics*. New York: Academic Press.

McFadzean, D., D. Stewart, and L. Tesfatsion. 2001. "A Computational Laboratory for Evolutionary Trade Networks." *IEEE Transactions on Evolutionary Computation* 5 (October): 546-560.

Meyer, David, Alexandros Karatzoglou, Friedrich Leisch, Christian Buchta and Kurt Hornik. "A Simulation Framework for Heterogeneous Agents." Computational Economics 22 (October): 285-301.

Miller, Preston J., ed. 1994. *The Rational Expectations Revolution: Readings From the Front Line*. Cambridge: MIT Press.

Mizuta, Hideyuki, Ken Steiglitz, and Erez Lirov. 2003. "Effects of Price Signal Choices on Market Stability." *Journal of Economic Behavior and Organization* 52 (October): 235-251.

Modigliani, Franco and Richard Brumberg. 1955. "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," in K.K. Kurihara ed. *Post-Keynesian Economics*. London: George Allen and Unwin.

Moretti, Sabrina. 2002. "Computer Simulation in Sociology: What Contribution?" *Social Science Computer Review* 20 (Spring): 43-57.

Mullineaux, Donald J. 1980. "Unemployment, Industrial Production, and Inflation Uncertainty in the United States." *Review of Economics and Statistics* 62 (May): 163-169.

Namatame, Akira. 2003. "Collective Intelligence with Selective Interactions and Reinforcement of Preferences." Conference paper at the 2003 Workshop on Collectives and the Design of Complex Systems.

Negroni, Giorgio. 2003. "Adaptive expectations coordination in an economy with heterogeneous agents." *Journal of Economic Dynamics and Control* 28 (October): 117-140.

North, Michael J. 2001. "Technical Note: Multi-Agent Social and Organizational Modeling of Electric Power and Natural gas Markets." *Computational and Mathematical Organization Theory Journal* 7 (December) 331-337.

___________. 2001. "Toward Strength and Stability: Agent-Based Modeling of Infrastructure Markets." *Social Science Computer Review* 19 (August) 307-323.

O'Donoghue, Cathal. 2001."Dynamic Microsimulation: A Methodological Survey." *Brazilian Electronic Journal of Economics* 4 (December).

Ostrom, Thomas M. 1988. "Computer Simulation: The Third Symbol System." *Journal of Experimental Psychology* 24 (September): 381-392.

Panzarasa, Pietro, Nicholas R. Jennings, and Timothy J. Norman. 2001. "Going Public and the Sale of Shares with Heterogeneous Investors: Agent-Based Computational Modelling and Computer Simulations." *Group Decision and Negotiation* 10 (September): 423-470.

Parsons, Simon. Piotr Gymtrasiewicz, and M. Wooldridge, eds. 2002. *Game Theory and Decision Theory in Agent-Based Systems*. New York: Springer.

Poterba, James M. 2001. "Demographic Structure and Asset Returns." *Review of Economics and Statistics* 83 (November): 565-584.

Potts, Jason. 2001. *The New Evolutionary Microeconomics: Complexity, Competence, and Adaptive Behaviour*. Cheltenham UK: Edward Elgar.

Pratt, John W. 1964. "Risk Aversion in the Small and in the Large." *Econometrica* 32 (January): 122-136.

Pryor, Richard, Grant Farnsworth, John Kelly, and Andrew Othling. 2002. *Successful Technical Trading Agents Using Genetic Programming*. SAND2004-4774. Albuquerque, NM: Sandia National Laboratories.

Pryor, R. J., D. Marozas, M. Allen, O. Paananen, K. Hiebert-Dodd, and R. K. Reinert. *Modeling Requirement for Simulating the Effects of Extreme Acts of Terrorism: A White Paper*. SAND1998-2289. Albuquerque, NM: Sandia National Laboratories.

Pylkkänen, Cathal. 2001. "Modeling Wages and Hours of Work." *Brazilian Electronic Journal of Economics* 4 (December).

Rivero, Sérgio Luiz de Medeiros, Bernd Heinrich Storb, Raul Sidnei Wazlawick. 1999. "Economic Theory, Anticipatory Systems and Artificial Adaptive Agents." *Brazilian Electronic Journal of Economics* 2 (June).

Romaniuk, Steve G. 1998. Evolutionary Agent Societies. In *Proceedings 4th International Conference on Information Systems Analysis and Synthesis*.

Sallach, David L. 2003. "Social Theory and Agent Architectures: prospective Issues in Rapid-Discovery Social Science." *Social Science Computer Review* 21 (Summer): 179-95.

Samuelson, Paul A. 1958. "An Exact Consumption-Loan Model of Interest with or without the Contrivance of Money." *Journal of Political Economy* 66 (6): 467-482.

Sapienza, Massimo Daniele, and Magda Fontana. 2002. "Search in Artificial Labour Markets: A Simulation Study," in Luna and Perrone eds. *Agent-Based Methods in Economics and Finance: Simulations in Swarm*. Boston: Kluwer Academic Publishers.

Sargent, Thomas J. 1973. "Rational Expectations, the Real Rate of Interest and the Natural Rate of Unemployment." Brookings Papers on Economic Activity. Washingtoin DC: The Brookings Institute.

___________, and Neil Wallace. 1973. "The Stability of Models of Money and Growth with Perfect Foresight." *Econometrica* 41 (November): 1043-1048.

Sayan, Serdar, and Ali Emre Uyar. 2002. "Directions of Trade Flows and Labor Movements between High- And Low-Population Growth Countries: An Overlapping Generations General Equilibrium Analysis" Bilkent University Department of Economics Discussion Paper No. 01-08 (January).

Schelling, Thomas C. 1978. *Micromotives and Macrobehavior*. New York: W. W. Norton & Company.

Schildt, Herbet. 2003. *C++ from the Ground Up, 3rd ed*. Berkley, CA: McGraw-Hill/Osborne.

Shea, John. 1995. "Union Contracts and the Life-Cycle/Permanent-Income Hypothesis." *The American Economic Review* 85 (March): 186-200.

Slepoy, Natasha A., and Richard J. Pryor. 2002. *Analysis of Price Equilibriums in the Aspen Economic Model under Various Purchasing Methods*. SAND2002-3693. Albuquerque, NM: Sandia National Laboratories.

Smith, Adam. 1937. *An Inquiry into the Nature and Causes of the Wealth of Nations*. New York: American Modern Library Series.

Spence, A. Michael. 1973. "Job Market Signaling." *Quarterly Journal of Economics* 87 (August): 355-374.

Spielauer, Martin, and Coomaren P. Vencatasawmy. 2001. "FAMSIM : Dynamic Microsimulation of Life Course Interactions between Education, Work, Partnership Formation and Birth in Austria, Belgium, Italy, Spain and Sweden." *Brazilian Electronic Journal of Economics* 4 (December).

Sprigg, James A., Jr. 2004. *Market Disruption, Cascading Effects, and Economic Recovery: A Life-Cycle Hypothesis Model*. SAND2004-5972. Albuquerque, NM: Sandia National Laboratories.

__________, Craig R. Jorgensen, and Richard J. Pryor. 2004. *Approach and Development Strategy for an Agent-Based Model of Economic Simulation*. SAND2004-4218. Albuquerque, NM: Sandia National Laboratories.

__________, and Mark A. Ehlen. 2004. *Full Employment and Competition in the Aspen Economic Model: Implications for Modeling Acts of Terrorism*. SAND2004-5500. Albuquerque, NM: Sandia National Laboratories.

__________, and Mark A. Ehlen. 2005. *Dynamic Nash Equilibrium in an Overlapping Generations Model: A Methodological Exercise in Economic Simulation*. SAND2005-2574-J. Albuquerque, NM: Sandia National Laboratories.

Sternman, John D. 2000. *Business Dynamics: Systems Thinking and Modeling for a Complex World*. New York: McGraw-Hill/Irwin.

Stiglitz, Joseph, and Jungyoll Yun. 2002. “Integration of Unemployment Insurance with Retirement Insurance.” NBER Working Paper 9199. Cambridge MA: National Bureau of Economic Research.

Teitelbaum, Daniel and Hadi Dowlatabadi. 2000. “A Computational Model of Technological Innovation at the Firm Level.” *Computational & Mathematical Organizational Theory* 6 (September): 227-247.

Terna, Pietro. 2002. “Cognitive Agents Behaving in a Simple Stock Market Structure,” in Luna and Perrone eds. *Agent-Based Methods in Economics and Finance: Simulations in Swarm*. Boston: Kluwer Academic Publishers.

Tesauro, Gerald, and Jeffrey O. Kephart. 2002. “Pricing in Agent Economies Using Multi-Agent Q-Learning.” *Autonomous Agents and Multi-Agent Systems* 5 (September): 289-304.

Tesfatsion, Leigh. 2001. “Economic Agents and Markets as Emergent Phenomena.” *Proceedings of the National Academy of Sciences* 99 (May): 7191-7192.

__________. 2002. “Agent-Based Computational Economics: Growing Economies From the Ground Up.” *Artificial Life* 8 (Winter): 55-82.

Thomas, W. H., M. J. North, and J. P. Peerenboom. 2004. “From Physics to Finances: Complex Adaptive Systems Representation of Infrastructure Interdependency.” *Naval Surface Warfare Center Technical Digest* 18: 18.

Tirole, Jean. 1985. “Asset Bubbles and Overlapping Generations.” *Econometrica* 53 (November): 1499-1528.

Train, Kenneth E.. 2003. *Discrete Choice Methods with Simulation*. UK: Cambridge University Press.

Virgo, John. 2001. "Economic Impact of the Terrorist Attacks of September 11, 2001." *Atlantic Economic Journal* 29 (December): 353-357.

Von Neumann, John, and Oskar Morgenstern. 1944. *Theory of Games and Economic Behavior*. Princeton: Princeton University Press.

Wallace, Neil. 1980. “The Overlapping Generations Model of Fiat Money,” in J. Kareken and N. Wallace (eds.) *Models of Monetary Economics*. Minneapolis: Federal Reserve Bank of Minneapolis.

Wang, Neng. “Precautionary Savings and Partially Observed Income.” *Journal of Monetary Economics* 51 (2004): 1645-1681.

Zott, Christopher. 2002. “When Adaptation Fails: An Agent-Based Explanation of Inefficient Bargaining Under Private Information.” *Journal of Conflict Resolution* 46 (December) 727-753.

Zovko, Ilija, and Farmer, J. Doyne. 2002. "The Power of Patience: A Behavioral Regularity in Limit-Order Placement." *Quantitative Finance* 2 (September): 387–392.

Distribution

1	0161	Patent & Licensing Office	11500
1	0619	Review & Approval Desk	09216 for DOE/OSTI
2	0899	Technical Library	09616
1	9018	Central Technical Files	08945-1
1	0378	W. J. Camp	01400
1	0378	J. E. Nelson	01430
1	0378	J. Strickland	01433
10	0196	G. A. Backus	01433
30	0318	R. Zalesak	01433